Math 264B Assignment 3 Due March 6, 2017 Remmel

1) Given a word $w = w_1 \dots w_n \in \{1,\dots,k\}^*$, let $|w| = n$ denote the length of w, $Z(w) = \prod_{i=1}^{n} z_{w_i}$, and $lev(w) = |\{i : w_i = w_{i+1}\}|$. Define a ring homomorphism ϕ on $\Lambda(x_1, x_2, \dots)$ by defining

$$\phi(e_n) = (-1)^{n-1} p_n(x_1,\dots,x_k)(x-1)^{n-1}$$

where p_k is the power symmetric function. Show that

$$\phi(\sum_{n\geq 0} h_n t^n) = \sum_{w\in\{1,\dots,k\}^*} x^{lev(w)} Z(w) t^{|w|}.$$

(2) Do problem 3.9 in the book.

(3) Do problem 3.10 in the book.

(4) Do problem 3.11 in the book.

(5) Do problem 3.16 in the book.

(6) Let $E_n^{(3)}$ denote the set of permutations $\sigma = \sigma_1 \dots \sigma_n \in S_n$ such that $\sigma_i > \sigma_{i+1}$ if and only if $i \equiv 0 \mod 3$. Find the generating funtion

$$1 + \sum_{n\geq 1} \frac{t^n}{n!} |E_n^{(3)}|.$$

(Hint: Modify the proof of the generating function for up-down permutations by finding separate expressions for the generating functions
$\sum_{n\geq 0} \frac{t^{3n}}{(3n)!} |E_{3n}^{(3)}|$,
$\sum_{n\geq 0} \frac{t^{3n+1}}{(3n+1)!} |E_{3n+1}^{(3)}|$, and
$\sum_{n\geq 0} \frac{t^{3n+2}}{(3n+2)!} |E_{3n+2}^{(3)}|$.