Math 152: Applicable Mathematics and Computing

May 15, 2017

Announcements

- Definition list for Midterm 2 has been posted on the website.

Last Time: The Prisoner's Dilemma

- Last time we saw the Prisoner's Dilemma.
- The payoff matrix was given by:

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (3,3) & (0,4) \\ (4,0) & (1,1) \end{pmatrix} \end{array}$$

Pure Strategic Equilibria

- **Def.** A pair of pure strategies x, y is a pure strategic equilibrium if

$$G_1(x, y) \geq G_1(x', y)$$

for any other pure strategy x' for Player I and

$$G_2(x, y) \geq G_2(x, y')$$

for any other pure strategy y' for Player II. G represents the payoff function of some bimatrix game.
- This is saying that x is the greatest entry in row y of Player I's payoff matrix and y is the greatest entry in row x of Player II's payoff matrix.

Pure Strategic Equilibria

- The Prisoner's Dilemma has one pure strategic equilibrium, corresponding to both players defecting.
- One of the desirable features of a strategic equilibrium is that it is self-enforcing: even without a binding agreement between players, it is in both player's interests to stay at a strategic equilibrium.

Pure Strategic Equilibria

- Pure strategic equilibria do **not** need to be unique:

$$\begin{pmatrix} (2,3) & (0,-3) \\ (-2,0) & (10,10) \end{pmatrix}$$

- And they do not necessarily exist:

$$\begin{pmatrix} (3,-3) & (0,0) \\ (1,-1) & (4,-4) \end{pmatrix}$$

Mixed Strategic Equilibria

- **Def.** A pair of (mixed) strategies $\mathbf{p}, \mathbf{q}$ is a strategic equilibrium if

$$G_1(\mathbf{p}, \mathbf{q}) \geq G_1(\mathbf{p}', \mathbf{q})$$

for any other strategy $\mathbf{p}'$ for Player **I** and

$$G_2(\mathbf{p}, \mathbf{q}) \geq G_2(\mathbf{p}, \mathbf{q}')$$

for any other strategy $\mathbf{q}'$ for Player **II**. G represents the payoff function of some bimatrix game.

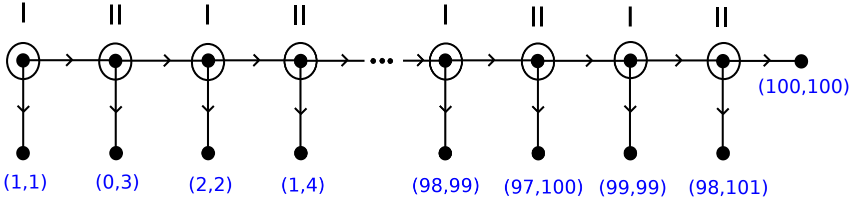
Dominated Strategies

- Let G be a bimatrix game with payoff matrices A and B for Players I and II respectively.
- **Def.** A row x' of A is dominated by row x if $A(x, y) \geq A(x', y)$ for all y.
- **Def.** A column y' of B is dominated by column y if $B(x, y) \geq B(x, y')$ for all x.
- For example:

$$\begin{pmatrix} (3,5) & (2,4) \\ (4,4) & (1,3) \end{pmatrix}$$

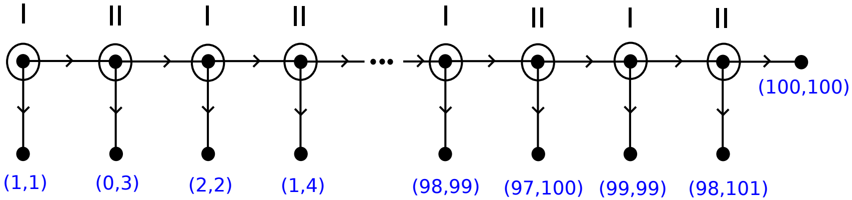
  Neither row dominates the other, but the first column dominates the second.
- **Remark.** Unlike for zero-sum games, to dominate a column we want to find another column that is *bigger than or equal* to it.

Centipede Game



What are the strategic equilibria for this game?

Centipede Game



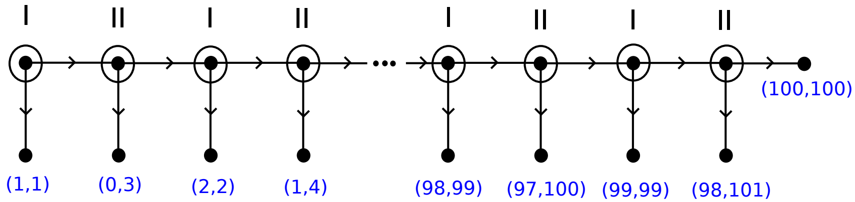
Well, note that in the right-most position that by domination Player II will always select "down" instead of "right".

Player I knows this. So Player I will select "down" not "right" in the second last position.

...

Continuing this, we eventually see that Player I chooses "down" on the first move.

Centipede Game



This is not a very satisfying answer. It does not match what we expect real players to do.

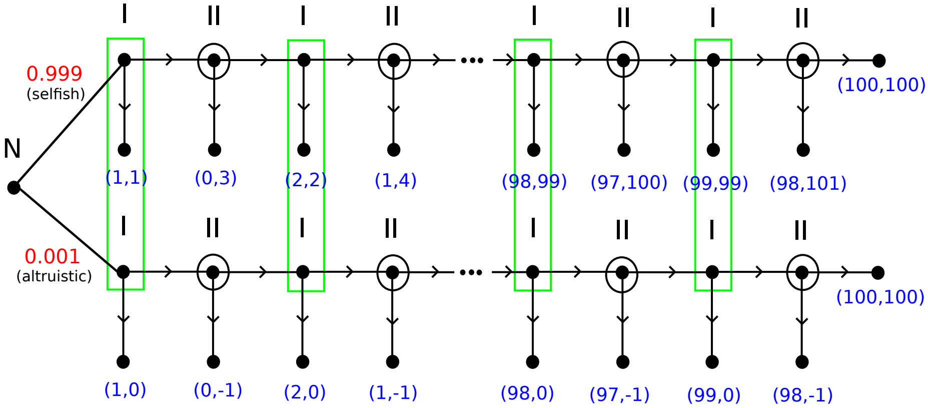
Centipede Game: Experimental Results

- If we were serious about this, the first step is to determine scientifically what real players actually do.
- McKelvey and Palfrey did just this in 1992 ("An Experiemental Study of the Centipede Game", *Ecnonometrica*, **60**(4) 803–836).
- In a 6-turn variant of the Centipede game, they found that only 1% of games ended on turn 1.
- *What is the value of our theory if it has no predictive power?*

Centipede Game: Experimental Results

- We should not be overly critical of the theory: we just need to work a bit harder.
- In reality, we know there is a chance our opponent will cooperate. So we are likely to cooperate a little ourselves.
- McKelvey and Palfrey realised this (and observed it in their experiments), so devised the following model: some proportion of people are altruists and will cooperate.
- When we start a game, we do not know if our opponent is an altruist. Let us put this information into our game.

Centipede Game: More Realistic Model



This game is harder to study, but it *does* have a strategic equilibrium where the players cooperate for many turns.

Strategic Equilibria Example

Game

Find the strategic equilibria, both pure and mixed, for the bimatrix game below.

$$\begin{pmatrix} (3,3) & (0,2) \\ (2,1) & (5,5) \end{pmatrix}$$

Strategic Equilibria Example

- It is easy to find the two pure strategic equilibria: the top left and bottom right entries.
- But what about mixed strategix equilibria?
- One useful approach is to look for an equalizing strategic equilibrium: try to find an equalizing strategy for each player on their *opponent's matrix*.
- **Why does this work?** Well, under such a strategy, your opponent's payoff is the same regardless of their move. So they have no incentive to switch (which is exactly a strategic equilibrium).

Strategic Equilibria Example

- For the example above, we can find an equalizing strategic equilibrium: $\mathbf{p} = \begin{pmatrix} 4/5 & 1/5 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 5/6 & 1/6 \end{pmatrix}$. The corresponding payoffs are $(5/2, 13/5)$.
- How do these three equilibria compare to the safety levels for this game?
- Can the payoffs in a strategic equilibrium ever be less than the safety levels? **No**. (Why?)