MA152 Spring 2017

Homework 1

Due: 12th April at 4PM in APM basement

1. In the subtraction game where players may subtract 1, 5 or 6 chips on their turn, identify the N and P positions.

2. One approach to studying Misère games is to first find the positions from which every move is to a terminal position, and label these as P positions. Now label all remaining positions using the algorithm we saw in class (the terminal positions do not need to be labelled).

   Consider the Misère subtraction game where a player may subtract 1, 3 or 4 chips. Using the above process, identify the N and P positions.

3. Consider a subtraction game, where on their turn a player removes 2^k chips, for some $k = 0, 1, 2, \cdots$ (that is, k is any non-negative integer). Identify the N and P positions in this game, with careful justification.

4. Explain why the Nim position $(14, 9, 7, 5, 2)$ is an N position. Find *all* winning moves from this position (a winning move is a move to a P-position).

5. The game of *Cheating Nim* follows the rules of Nim, with a small modification. There is a pile of 100 *spare chips* under the table, and on their turn a player may, instead of the usual rule of removing chips from a pile, take some number of chips from under the table and add them to one of the piles on the table. If there are no remaining chips under the table, the game proceeds just as in normal Nim. Like in normal Nim, when chips are removed from a pile on the table they are removed from the game.

   Show that the N and P positions in this game are the same as those in normal Nim.

6. Consider the subtraction game where on the first turn a player may remove any number of chips, except they cannot remove the whole pile, and must remove at least one chip. On subsequent turns, the player may remove any number between 1 and x chips, where x is is the number of chips removed by their opponent on the previous turn.

   The positions in this game can be denoted by pairs of integers (n, m), where n is the number of chips, and m is the maximum number of chips a player may remove.

   (a) List all moves from the position $(50, 4)$.

   (b) For all positions of the form $(n, 1)$, describe the P and N positions.

   (c) For all positions of the form $(n, 2)$, describe the P and N positions.

   (d) Find an optimal move for the first player when the game starts with $n = 44$ chips (that is, from the position $(44, 43)$).

   (e) In general, which positions (n, m) are P positions?