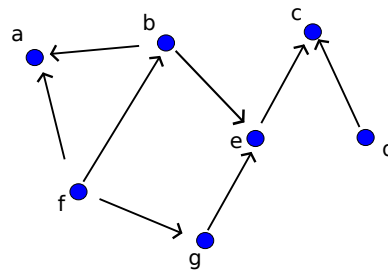# MA152 Spring 2017

Homework 2

Due: 19th April at 4PM in APM basement

1. For the subtraction game, where on their turn a player may remove 2 or 3 chips, determine the Sprague–Grundy function for all positions.

2. Compute the Sprague–Grundy function of all positions of the directed graph in the figure.


3. Let G_1 be the subtraction game where on their turn a player may remove 1 or 2 coins, and where there are 10 coins initially. Let G_2 be the game of Nim with three piles, of sizes $1, 6, 7$. List all winning moves in $G_1 + G_2$.

4. Consider a game where there are piles of chips, and on a player's turn they take any pile with more than 1 chip and divide this pile into two new piles, each with at least 1 chip. The terminal position is where all piles have exactly one chip. Find the Sprague–Grundy function of this game for all games with a single pile, where there are at most 10 chips.

5. The game of *Tic-Tac-Tic* is played on a 3×3 board, like Tic-Tac-Toe, except this game is impartial. On each player's turn, they place an 'O' in any one of the empty squares. The game ends if there are three 'O's

in any row, column or diagonal. The last player to make a move is the winner.

(a) Describe the terminal positions in this game (do *not* list all of them).

(b) Show that the position where no 'O's have been placed yet, pictured in the figure below, is an N-position.
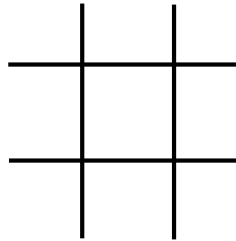
(c) Determine whether the position in the figure below is an N or a P-position.
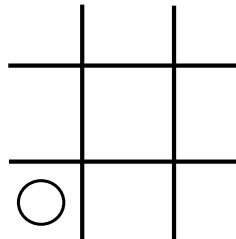
6. In the game of *Turning Turtles*, some coins are placed in a line on a table, from left to right. Each coin has two sides, called *heads* (H) and *tails* (T). The coins are placed on the table so that one side is visible, and the other side is facing downwards. We describe a position by writing down for each coin which side is visible (either 'H' or 'T'). For example, "THTTT" denotes the position where there are 5 coins on the table, the leftmost coin is showing the 'T' side, the next coin is showing 'H' and the three coins to the right of this are showing 'T'.

A move consists of: flipping exactly one 'H' into a 'T', and then (if the player wishes) flipping any coin to the left of this (this second flip can either be from 'H' to 'T' or from 'T' to 'H'). The terminal positions in the game are the positions where all coins are showing 'T'.

(a) List all moves from the position "HTHTT".

(b) Find the P-positions among all positions where there is exactly one 'H' (and all remaining coins are showing 'T').

(c) Identify all P-positions.