MA152 Spring 2017

Homework 3

Due: 3rd May at 4PM in APM basement

1. Solve the following matrix game (that is, find the value and an optimal strategy for each player):

$$\begin{pmatrix} 5 & -1 & -2 \\ -3 & 10 & 5 \\ -2 & 7 & 6 \end{pmatrix}$$

2. Consider a two-player game where both players simultaneously announce either the number 1 or the number 2. If both numbers are equal then Player I wins, and if both numbers are different then Player II wins. The payoff is the product of the two numbers. Find the value of the game and a pair of optimal strategies.

3. Solve the games:

   (a)
   $$\begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$$

   (b)
   $$\begin{pmatrix} 3 & 0 \\ -1 & 5 \end{pmatrix}$$

4. Solve the following matrix game (that is, find the value and an optimal strategy for each player):

$$\begin{pmatrix} 3 & 2 & 4 & 0 \\ -2 & 1 & -4 & 5 \end{pmatrix}$$

5. Consider a two-player game where both players simultaneously announce 1, 2 or 3. If the sum of the numbers is odd then Player I wins, otherwise Player II wins. The payoff is the sum of the two numbers. Assume that the optimal strategy for Player II assigns positive probability to every column. Use the principle of indifference to find the value of the game and optimal strategies for both players.

6. In "Normandy: Game and Reality" by W. Drakert, an analysis is given of the Allied invasion of Normandy (D-Day). Six possible attacking configurations (1-6) for the Allies were given, and six possible defensive strategies (A-F) for the Germans. All 36 pairs were evaluated and simulated. The following table gives a numerical estimate of the value of each situation to the Allied forces.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 13 & 29 & 8 & 12 & 16 & 23 \\ 18 & 22 & 21 & 22 & 29 & 31 \\ 18 & 22 & 31 & 31 & 27 & 37 \\ 11 & 22 & 12 & 21 & 21 & 26 \\ 18 & 16 & 19 & 14 & 19 & 28 \\ 23 & 22 & 19 & 23 & 30 & 34 \end{pmatrix} \end{array}$$

Find the value of this game, and a pair of optimal strategies. The Allies and Germans opted for strategies 1 and B respectively, were these good choices?