MA152 Spring 2017

Homework 4

Due: 10th May at 4PM in APM basement

1. Solve the following matrix game (that is, find the value and an optimal strategy for each player):

$$\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

2. Solve the following matrix game:

$$\begin{pmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. Using invariance, solve the game:

$$\begin{pmatrix} -4 & 1 & 2 \\ 1 & -5 & 1 \\ 2 & 1 & -4 \end{pmatrix}$$

4. Consider the game where simultaneously Player I announces an integer x and Player II announces an integer y, where $1 \leq x, y \leq 1000$. If $x \geq y$ then Player I wins $x - y$, otherwise Player II wins $y - x$. What is the value of this game?

5. Consider the game

$$\begin{pmatrix} 3 & 1 \\ 1 & 5 \end{pmatrix}$$

(a) If Player I knows that Player II's mixed strategy is $(q \quad 1-q)^T$, find Player I's best response strategy (the answer will depend on q).

(b) In this situation, what value of q should Player II choose to minimize her losses?

6. (a) Let A be a matrix game with value V. Let B be the matrix game that is obtained by adding a constant c to every entry of A. Briefly justify why the value of B is $V + c$.

   (b) Using (a), find the value of the game

$$\begin{pmatrix} 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \\ 5 & 4 & 3 & 2 \end{pmatrix}$$