MA152 Spring 2017

Homework 7

Due: 7st June at 4PM in APM basement

1. Draw the TU and NTU-feasible sets for the following bimatrix game. Indicate the Pareto optimal curve in both diagrams.

$$\begin{pmatrix} (0,4) & (3,2) \\ (4,0) & (2,3) \end{pmatrix}$$

2. Find the TU solution and sidepayment for the bimatrix game

$$\begin{pmatrix} (3,1) & (4,3) & (-5,-5) \\ (0,5) & (1,0) & (5,0) \end{pmatrix}$$

3. For each of the following bimatrix games, find the NTU solution given that the threat point is $(0,0)$ (use the Nash approach, and not λ-transfer).

   (a)

$$\begin{pmatrix} (1,5) & (0,0) \\ (1,1) & (3,0) \end{pmatrix}$$

   (b)

$$\begin{pmatrix} (1,5) & (0,0) \\ (0,0) & (2,4) \end{pmatrix}$$

4. Let $S = \{(x,y) : 0 \leq y \leq 4 - x^2\}$ be an NTU-feasible set.

   (a) Find the NTU solution if the threat point is $(u^*, v^*) = (0,0)$.

   (b) Find the NTU solution if the threat point is $(u^*, v^*) = (0,1)$.

5. (*Not to be handed in.*) Consider a three-player game with the following characteristic function: $v(\{1\}) = 1$, $v(\{2\}) = 0$, $v(\{3\}) = 2$, $v(\{1,2\}) = 2$, $v(\{2,3\}) = 3$, $v(\{1,3\}) = 4$, $v(\{1,2,3\}) = 7$. Compute the Shapeley values for each player.

6. (*Not to be handed in.*) Consider the following 3-person game of perfect information. Let $S = \{1, 2, \cdots, 10\}$. First Player 1 chooses $i \in S$. Then Player 2, knowing i, chooses $j \in S$, $j \neq i$. Finally Player 3, knowing i and j, chooses $k \in S$, $k \neq i$, $k \neq j$. The payoff given these three choices is $(|i-j|, |j-k|, |k-i|)$. Find the coalitional form of the game.

7. (*Not to be handed in.*) Consider the three player game where each player simultaneously announces 0 or 1. Let x be the sum of the three announced numbers. If x is a multiple of 3, the payoff is x to Player 1 and 0 to the other players. If x is 1 more than a multiple of 3, the payoff is x to Player 2 and 0 to the other players. Finally if x is 2 more than a multiple of 3, the payoff is x to Player 3 and 0 to the other players. Find the coalitional form of the game.