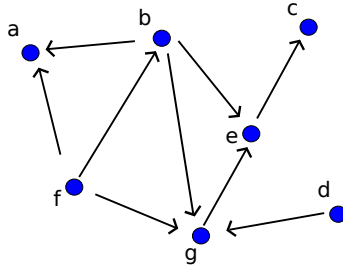


MA152 Midterm 1

Provide justification in your answers.

21st April 2017

- State the formal definition of N and P-positions (also called the *characteristic property*).
 - For the subtraction game, where players may remove 2, 3 or 5 chips on their turn, determine which positions are N-positions and which are P-positions.
- For a progressively bounded, directed graph G , state the definition of the Sprague–Grundy function g .
 - For the graph game below, compute the Sprague–Grundy value for every position.



- Consider a game where there are piles of chips, and on a player's turn they may either: remove any positive number of chips from a single pile, **or** divide any single pile into exactly two new piles. Compute the Sprague–Grundy value for this game for positions that consist of a single pile, with between 0 and 12 chips.
- The game of *Kayles* is played with a set of coins arranged in a line, where each coin has two sides: 'H' and 'T'. On a player's turn, they may either flip one 'H' into a 'T', or they may take two adjacent 'H's and flip both into 'T's. For the position below, determine all winning moves (ie. moves to P-positions):

“HHHHHHHHHHHTHHHH”
(ie. 11 'H's, 1 'T' then 4 'H's)

The last player to make a move is the winner (the *normal play* rule).
(*Hint*: Translate the game into a game with piles of chips.)