## MA152 Midterm 2

## Provide justification in your answers.

19th May 2017

1. (a) State the definition of the value of a two-person zero sum game.
(b) For the $4 \times 4$ matrix game below, find the value and an optimal strategy for each player.

$$
\left(\begin{array}{rrrr}
1 & -1 & 2 & -4 \\
0 & 1 & -1 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 1
\end{array}\right)
$$

2. (a) Define what it means for one row to dominate another.
(b) Find the value of the $4 \times 4$ matrix below, and an optimal strategy for each player.

$$
\left(\begin{array}{rrrr}
2 & -1 & 1 & 2 \\
0 & 5 & 1 & 3 \\
2 & -2 & 0 & 1 \\
1 & 6 & 4 & 1
\end{array}\right)
$$

3. Player I and Player II play a game where they each simultaneously announce an integer between 1 and 4 (inclusive). Let $x$ be the number chosen by Player I, and let $y$ be the number chosen by Player II. If $x \leq y$, then Player I wins. Otherwise, Player II wins. The losing player pays $x y$ (ie. the product of the two numbers) to the winning player. Construct the payoff matrix, and then find the value of the game, and an optimal strategy for each player.
4. For the game in Extensive Form below, convert it to Strategic Form, and then find the value of the game.

