Practice questions for midterm 2

Section 10.1

1. Find the domain of a given vector function $\mathbf{r}(t)$.

   **Sample Question**

   Find the domain of the vector function

   $$\mathbf{r}(t) = (\sqrt{1-t}, t, \ln(t))$$

2. Sketch curve given by a vector function $\mathbf{r}(t)$.

   **Sample Question**

   Sketch the curve given by the vector function

   $$\mathbf{r}(t) = (\cos(t), t)$$

   Indicate with an arrow on your sketch the direction of increasing $t$.

3. Find the point of intersection (between space curve given by $\mathbf{r}(t)$ and a surface).

   **Sample Question**

   Find the points at which the space curve given by

   $$\mathbf{r}(t) = t\mathbf{i} + (2t - t^2)\mathbf{k}$$

   intersects the paraboloid $z = x^2 + y^2$.

4. Find the curve of intersection between two surfaces.

   **Sample Question**

   Find the vector function representing the curve of intersection of the cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$.

5. Two particle trajectories: do they collide? do they cross paths?

   **Sample Question**

   The trajectories of two particles are given by the vector functions

   $$\mathbf{r}_1(t) = (t^2, 7t - 12, t^2) \quad \mathbf{r}_2(t) = (4t - 3, t^2, 5t - 6)$$

   for $t \geq 0$.

   Do the particles collide? Do their paths intersect?
Section 10.2

1. Given $r(t)$, find $r'(t)$.

   **Sample Question**
   Find the derivative of the vector function
   \[ r(t) = \langle e^{t^2}, -1, \ln(1 + 3t) \rangle \]

2. Find tangent line to a given curve.

   **Sample Question**
   Find parametric equations for the tangent line to the curve with the parametric equations
   \[
   \begin{align*}
   x &= 1 + 2\sqrt{t} \\
   y &= t^3 - t \\
   z &= t^3 + t
   \end{align*}
   \]
   at the point $(3, 0, 2)$.

3. Integration of a vector function.

   **Sample Question**
   Evaluate
   \[
   \int_0^1 \left( 3t^{-1/2} \mathbf{i} + \sin(2t) \mathbf{j} - e^{-t} \mathbf{k} \right) \, dt
   \]

Section 11.1

1. Find/sketch the domain of a multivariable function $f(x, y)$.

   **Sample Question**
   Let $f(x, y) = \ln(y - 2x - 3)$.
   (a) Evaluate $f(1, 1)$
   (b) Find and describe the domain of $f$
2. Draw a contour map of a given function $f(x, y)$.

**Sample Question**

Draw a contour map of the function $f(x, y) = \sqrt{x} + y$, showing several level curves.

**Section 11.3**

1. Find the first-order partial derivatives of a given function.

**Sample Question**

Find the first order partial derivatives of the following functions

(a) $f(x, y) = y^5 - 3xy$
(b) $w = \ln(x + 2y + 3z)$
(c) $z = (2x + 3y)^{10}$

2. Implicit differentiation with partial derivatives

**Sample Question** $z$ is defined implicitly as a function of $x$ and $y$ by the relation

$$x^2 + y^2 + z^2 = 3xyz$$

Use implicit differentiation to find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

3. Find the second-order partial derivatives of a given function.

**Sample Question** Find all the second-order partial derivatives of

$$f(x, y) = x^3y^5 + 2x^4y$$

Verify that Clairaut’s theorem holds for this function.


**Sample Question** If $f(x, y, z) = xy^2z^3 + \arcsin(x\sqrt{z})$, find $f_{xyz}$.

**Hint:** Which order of integration is easiest?

**Section 11.4**

3
1. Find the equation of the tangent plane to a given surface (at a given point).

**Sample Question** Find an equation of the tangent plane to \( z = 3y^2 - 2x^2 + x \) at \((2, -1, -3)\).

2. Find linearization of a given function (at a given point). Use the linearization to estimate function value.

**Sample Question** Explain why the function \( f(x, y) = x\sqrt{y} \) is differentiable at \((1, 4)\) and find the linearization \( L(x, y) \) of \( f(x, y) \) at that point.

Use \( L(x, y) \) to estimate \( f(0.9, 4.1) \), do you expect this to be a good estimate?

Could \( L(x, y) \) be used to estimate \( f(3.1, -0.1) \)?