0. (1 point.) Carefully read and follow the instructions.

1. (6 points.) Find all critical points of the function

\[ f(x, y) = 2y^3 - 6xy + 3x^2 \]

and classify each of them as a local maximum, a local minimum, or a saddle point.

2. (a) (4 points.) Calculate the tangent plane to the graph of the function

\[ f(x, y) = xe^{x+y^2} \]

at the point \((-1, 1)\).

(b) (2 points.) Use linear approximation to estimate \(f(-1.1, 0.8)\).
3. (a) (4 points.) A farmer from Omaha, Nebraska is tending to his corn field in the dead of winter. Unfortunately he forgot his mittens and is freezing. The temperature in the field is given by the function \( T(x, y) = \frac{1}{2x^2 + 1y^2} \) where \( x \) is measured in latitide and \( y \) is measured in longitude. If the farmer is standing at the point \((a, b) = (-1, -2)\), in which direction should he travel to warm up the fastest? Your answer should be a vector in 2 dimensions. 

Note: this function is not actually a realistic depiction of Temperature!

(b) (2 points.) Sadly, this farmer never took Calc III and has no idea what to do. He ends up walking in a straight line towards the point \((-3, -1)\). Is the farmer warming up or getting colder? How do you know?

4. (6 points.) Suppose \( f(x, y) = x^2 - xy \) and \( x(s, t) = \frac{t}{s}, y(s, t) = s^2 + t \). Using the multivariable chain rule calculate the partial derivatives

\[
\frac{\partial f}{\partial s} \bigg|_{(s,t)=(2,1)} \quad \frac{\partial f}{\partial t} \bigg|_{(s,t)=(2,1)}
\]

WARNING: answers that do not use the chain rule will receive no credit.

5. (extra credit: 1 pt.) Is it possible for a differentiable function \( g(x, y) \) to have the following partial derivatives?

\[
g_x(x, y) = xe^y \cos(x) \quad g_y(x, y) = ye^x \cos(y)
\]

Explain.