MATH 170B HW4

6.1

#2 Proof: ① By Thm I, Lf is unique, and Langrange form is that unique polynomial, thus

$$Lf = \Sigma f(x_i) l_i.$$

② Show L is linear.

$$L(af+bg) = \cancel{aLf+bLg}$$

$$\Sigma (af+bg)(x_i) l_i = a\Sigma f(x_i) l_i + b\Sigma g(x_i) l_i$$

$$= aLf + bLg.$$

Ω.

#4. Proof: By Thm I, Lq is the unique polynomial at order n most s.t. $Lq(x_i) = q(x_i)$.

And q itself is $q(x_i) = q(x_i)$,

thus, q is the unique polynomial, i.e. $Lq = q$.

Ω.

#5. Proof: Consider $f(x) = 1$, then

$$Lf = \sum_{i=0}^{n} f(x_i) l_i(x) = \sum_{i=0}^{n} l_i(x).$$

By #4, $Lf = f$, thus $\sum_{i=0}^{n} l_i(x) = 1$.

Ω

#6 proof:

$$\sum_{i=0}^{n} [f(x)-f(x_i)]l_i(x) = f(x)\sum_{i=0}^{n} l_i(x) - \sum_{i=0}^{n} f(x_i)l_i(x) \quad \cdots (1)$$

then by #2 and #5

$(1) = f(x) - p(x)$. □

#21

x	2	0	3
$f(x)$	11	7	28

① Lagrange form:

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-0)(x-3)}{(2-0)(2-3)} = -\frac{1}{2}x(x-3)$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-2)(x-3)}{(0-2)(0-3)} = \frac{1}{6}(x-2)(x-3)$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-2)(x-0)}{(3-2)(3-0)} = \frac{1}{3}(x-2)x$$

$$\Rightarrow p(x) = -\frac{11}{2}x(x-3) + \frac{7}{6}(x-2)(x-3) + \frac{28}{3}x(x-2).$$

② Newton's form:

$$p(x) = c_0 + c_1(x-2) + c_2(x-2)(x-0)$$

Then solve $\begin{cases} p(x_0) = 11 \\ p(x_1) = 7 \\ p(x_2) = 28 \end{cases} \Rightarrow \begin{cases} c_0 = 11 \\ c_0 + c_1(0-2) = 7 \\ c_0 + c_1(3-2) + c_2(3-2)(3-0) = 28 \end{cases}$

$$\Rightarrow \begin{cases} c_0 = 11 \\ c_1 = 2 \\ c_2 = 5 \end{cases} \Rightarrow p(x) = 11 + 2(x-2) + 5(x-2)x.$$

#22. The same as #21.

6.2 #5. proof: According to C(10) on page 329,

$$\sum_{i=0}^{n} p[x_0, x_1, \cdots, x_i] \prod_{j=0}^{i-1} (x-x_j)$$ is the newton's form.

By #4 in 6.1, $p(x) = Lp = \sum_{i=0}^{n} p[x_0 \cdots x_i] \prod_{j=0}^{i-1} (x-x_j)$. ☐.

#8. proof: By Thm 1, Lagrange form = Newton's form ☐.

#24.

x	4	2	0	3
$f(x)$	63	11	7	28

$p(x) = c_0 + c_1(x-4) + c_2(x-4)(x-2) + c_3(x-4)(x-2)x$

	x	$f(x)$			
x_0	4	63	26	6	1
x_1	2	11	2	5	
x_2	0	7	7		
x_3	3	28			

upper trianglar table

$c_0 = 63$

$c_1 = 26$

$c_2 = 6$

	x	$f(x)$			
x_0	4	63			
x_1	2	11	26		
x_2	0	7	2	6	
x_3	3	28	7	5	1

Lower triaglur table

$c_3 = 1$.

6.3 #1.

x	0	1	2
$p(x)$	2	-4	44
$p'(x)$	-9	4	X

Follow these arrows in data, one has

$$p(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^2(x-1) + c_4 x^2(x-1)^2.$$

	x	$f(x)$				
x_0	0	2	$p'(0)=-9$	3	7	5
x_0	0	2	-6	10	17	
x_1	1	-4	$p'(1)=4$	44		
x_1	1	-4	48			
x_2	2	44				

upper

$\Rightarrow$

$c_0 = 2$

$c_1 = -9$

$c_2 = 3$

$c_3 = 7$

$c_4 = 5$

	x	$f(x)$				
x_0	0	2				
			-9			
x_0	0	2				
			-6	3		
x_1	1	-4				
			4	10	7	
x_1	1	-4				5
			48	44	17	
x_2	2	44				

lower

6.4 #21 Natural spline requires

$$S''(-1) = S''(1) = 0.$$

Since $S_1''(x) = 6x \Rightarrow S''(-1) = -6 \neq 0$

Thus not a ~~spline~~ Natural spline.

#26. Since $\begin{array}{l} S_1(1) = S_2(1) \\ S_1'(1) = S_2'(1) \\ S_1''(1) = S_2''(1) \end{array} \Rightarrow \begin{cases} c = 1 \\ b = 3 \\ 2a = 6 \end{cases} \Rightarrow \begin{cases} a = 3 \\ b = 3 \\ c = 1 \end{cases}$

$$\Rightarrow f(x) = \begin{cases} x^3 & , [0,1] \\ \frac{1}{2}(x-1)^3 + 3(x-1)^2 + 3(x-1) + 1 & , [1,3] \end{cases}$$

Then, since $S_1''(0) = 0$

$$S_2''(3) = 3(x-1) + 2a \neq 0,$$

thus not a natural spline.