

HW 2

Math 17B

$$\begin{aligned}
 2.4 \text{ (a). } f(x) &= \frac{1}{2} F(x)^T F(x) \\
 &= \frac{1}{2} \begin{pmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{pmatrix}^T \begin{pmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{pmatrix} \\
 &= \frac{1}{2} (f_1(x)^2 + \dots + f_n(x)^2)
 \end{aligned}$$

then ~~$\nabla f(x) = \frac{1}{2}$~~

$$\frac{\partial f(x)}{\partial x_1} = \frac{1}{2} \left(2f_1 \frac{\partial f_1}{\partial x_1} + \dots + 2f_n \frac{\partial f_n}{\partial x_1} \right)$$

$$\Rightarrow \nabla f = \begin{pmatrix} f_1 \frac{\partial f_1}{\partial x_1} + \dots + f_n \frac{\partial f_n}{\partial x_1} \\ \vdots \\ \cancel{f_1 \frac{\partial f_1}{\partial x_n}} + \dots + f_n \frac{\partial f_n}{\partial x_n} \\ f_1 \frac{\partial f_1}{\partial x_n} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \dots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}^T \begin{pmatrix} f_1 \\ \vdots \\ f_n \end{pmatrix} = F'(x)^T F(x)$$

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$$(b). f(x+\lambda y) = f(x) + \lambda y^T \nabla f(x) + \lambda^2 y^T \nabla^2 f(x) y.$$

Since $y^T \nabla f(x) < 0$, thus, we can always find a small enough λ , s.t.

$$\lambda y^T \nabla f(x) + \lambda^2 y^T \nabla^2 f(x) y < 0$$

$$\Rightarrow f(x+\lambda y) < f(x)$$

□

(c). To show Newton's direction is always a direction of decrease for f at x .

Only need to show it is a descent direction. then it is a direction of decrease by (b).

$$\begin{aligned} \text{Consider } y^T \nabla f(x) &= -F^T F'^{-T} F'^T F \\ &= -F^T I F = -F^T F < 0 \end{aligned}$$

i.e. y is a descent direction.

(d). Assume we have error with $y = -F'(x)^{-1} F(x) + E(x)$.

To make sure y is still a descent direction,

$$\text{One need } y^T \nabla f(x) = -F^T F + E^T F'(x)^T F(x) < 0$$

$$\text{i.e. } E^T F'(x)^T F(x) \ll F^T F = \|F(x)\|^2$$

Otherwise, Newton's method doesn't work.

2.5: The entire proof is on page 62
above equation (2.6.4).

Remark: In matlab, to call a function in another
function file, need to add "@" in front.

For example:

In Fcn1.m,

To call Fcn2 in Fcn1.m,

use `feval(@Fcn2, inputs - Fcn2);`

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hw2.pdf

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