MATH171B Hw4.

#4.1. Proof:

① Let $x = x^*$, one has, $\forall y$,

$$f(y) \geq f(x^*) + f(x^*)(y - x^*) = f(x^*)$$.

thus x^* is a global minimizor.

② Let $x = x^*$, $\forall y$

$f(y) > f(x^*)$. Thus, x^* is a global minimizor.

Assume $\exists\, \bar{x} \neq x^*$ is a global min.

then $f(\bar{x}) > f(x^*)$ contradiction. Hence x^* is unique.

#4.2.
(a). $$g(x) = \begin{pmatrix} 2x_1 \\ 2x_2\cos x_3 - e^{x_2} x_3^2 \\ -x_2^2 \sin x_3 - 2e^{x_2} x_3 + 4 \end{pmatrix}$$

$$H(x) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2\cos x_3 - e^{x_2} x_3^2 & -2x_2 \sin x_3 - 2e^{x_2} x_3 \\ 0 & -2x_2 \sin x_3 - 2e^{x_2} x_3 & -x_2^2 \cos x_3 - 2e^{x_2} \end{pmatrix}$$

(b). $H(\bar{x}) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1-2e \end{pmatrix} \approx \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -6.4366 \end{pmatrix}$

$\Rightarrow H(\bar{x}) = I \begin{pmatrix} 2 & & \\ & 2 & \\ & & -6.4366 \end{pmatrix} I^T = I D I^T$.

(c). Solve $\quad H(\bar{x}) p^N = -g(\bar{x}) = \begin{pmatrix} 0 \\ -2 \\ -4 \end{pmatrix}$

$\Rightarrow p^N = \begin{pmatrix} 0 \\ -1 \\ 0.6214 \end{pmatrix}$, test ${p^N}^T g(\bar{x}) = 0.4858 > 0$
Not a desent direction.

(d). Let $\bar{D} = \begin{pmatrix} 2 & & \\ & 2 & \\ & & 6.4366 \end{pmatrix}$

$\Rightarrow B = I \bar{D} I^T = \begin{pmatrix} 2 & & \\ & 2 & \\ & & 6.4366 \end{pmatrix}$

Solve $\quad B p^M = -g(\bar{x}) \Rightarrow p^M = \begin{pmatrix} 0 \\ -1 \\ -0.6214 \end{pmatrix}$

The $\quad \nabla_{p^M} f(\bar{x}) = g(\bar{x}) \cdot \frac{p^M}{\|p^M\|_2} = -3.81$.

(e). Let $p = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, then $p^T H(\bar{x}) p < 0$, i.e. p is a direction of negative curvature.