MATH 174/274 HW1

$\boxed{0.5.1}$

(a) Obviously $f(x)$ is continuous on $[0,1]$

Since $f(0)=1$, $f(1)=-2$,

By Intermediate Value Theorem, there must exist

a $c \in [0,1]$, s.t. $f(c)=0$ $\square$

(b). Obviously $f(x)$ is continuous on $[0,1]$

Since $f(0)=1$, $f(\frac{1}{2})=-\frac{1}{2}$

By IVT, there must exist $c \in [0,\frac{1}{2}]$, s.t. $f(c)=0$. $\square$

$\boxed{0.5.2}$

(a) $\begin{cases} \cancel{f(a)-f(b)} = f(b)-f(a) = f(1)-f(0) = e-1 \\ b-a = 1-0 = 1 \end{cases}$

Need to find c, s.t. $f'(c) = e-1/1$

Since $f'(x) = e^x$,

Solve $f'(c) = e^c = e-1$

$\Rightarrow \quad e(1-e^{c-1}) = 1$

$\Rightarrow \quad e^{c-1} = 1-e^{-1}$

$\Rightarrow \quad c = \ln(1-e^{-1})+1$

then let $c = \ln(1-e^{-1})+1$

Check $\ln(1-e^{-1})+1$
$\approx 0.54 \in [0,1]$.

(c) $f(x) = 1/x+1$

$$\begin{cases} f(b)-f(a) = f(1)-f(0) = -\frac{1}{2} \\ b-a = 1-0 = 1 \end{cases},$$

Need to find c s.t. $f'(c) = -\frac{1}{2}$

Since $f'(x) = -(x+1)^{-2}$

Solve $-(c+1)^{-2} = -\frac{1}{2}$

$\Rightarrow (c+1)^2 = 2$

$\Rightarrow c = \sqrt{2}-1$

Check $\sqrt{2}-1 \in [0,1]$

then let $c = \sqrt{2}-1$.

$\boxed{0.5.3}$

(a) Solve $\int_0^1 x \cdot x \, dx = c\int_0^1 x\,dx$

$\Rightarrow \frac{1}{3}x^3\Big|_0^1 = c/2 \, x^2\Big|_0^1$

$\Rightarrow \frac{1}{3} = \frac{c}{2} \Rightarrow c = 2/3$

(c) Solve $\int_0^1 xe^x dx = c\int_0^1 e^x dx$

$\Rightarrow (xe^x - e^x)\Big|_0^1 = ce^x\Big|_0^1$

$\Rightarrow 1 = c(e-1)$

$\Rightarrow c = 1/e-1$.

$\boxed{0.5.4}$

(a) $P_2(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2, \quad x_0 = 0$

$= 1 + e^{x^2}\cdot 2x\big|_{x=0}\cdot x + \frac{(e^{x^2}\cdot 4x^2 + e^{x^2}\cdot 2)}{2}\Big|_{x=0}\cdot x^2$

$= 1 + x^2$

(c) $P_2(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2, \quad x_0 = 0$

$= 1 + (-(x+1)^{-2})\big|_{x=0}\cdot x + \frac{(2(x+1)^{-3})}{2}\Big|_{x=0}\cdot x^2$

$= 1 - x + x^2$

$\boxed{1.1.1}$

(a) ~~Test $f(2) = 2^3$~~ Let $f(x) = x^3 - 9$

Test $f(2) = -1$, $f(3) = 18$

By IVT, there exists a root in $[2, 3]$.

(c) Let $f(x) = \cos^2 x - x + 6$

Test $f(6) > 0$, $f(7) < 0$

By IVT, there exists a root in $[6, 7]$.

$\boxed{1.1.3}$ $f(x) = x^3 - 9$

(a) First, check $f(2) \cdot f(3) < 0$

Then, apply Bisection method,

k	a_k	$f(a_k)$	c_k	$f(c_k)$	b_k	$f(b_k)$
0	2	$-$	2.5	$+$	3	$+$
1	2	$-$	2.25	$+$	2.5	$+$
2	2		$\boxed{2.125}$		2.25	

$\hookrightarrow$ $c_2 = 2.125$.

(c) $f(x) = \cos^2 x - x + 6$

First, check $f(6) \cdot f(7) < 0$

Then, apply Bisection method,

k	a_k	$f(a_k)$	c_k	$f(c_k)$	b_k	$f(b_k)$
0	6	$+$	6.5	$+$	7	$-$
1	6.5	$+$	6.75	$+$	7	$-$
2	6.75	$+$	$\boxed{6.875}$		7	$-$

$\hookrightarrow$ $c_2 = 6.875$

$\boxed{1.1.5}$

(a) Define $f(x) = x^4 - x^3 - 10$

Consider root finding Problem $f(x) = 0$

Since $f(2) = -2 < 0$

$f(3) = 44 > 0$

So, inside $[2, 3]$, there exists a solu.

(b). By equ (1.1) on page 28

Error $= |x_c - r| < \frac{b-a}{2^{n+1}}$, where n is steps needed.

In order to keep the error less then 10^{-10},

Solve $\frac{b-a}{2^{n+1}} = \frac{1}{2^{n+1}} \leq 10^{-10}$

$\Rightarrow \quad 2^{n+1} > 10^{10}$

$\Rightarrow \quad n > {}^{10}/_{\lg 2} - 1 > 32.2193$

then 33 steps needed.

$\boxed{1.2.1}$ (a) Solve $x = {}^{3}/_{x} \Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$

(c) Solve $x^2 - 4x + 2 = x \Rightarrow x^2 - 5x + 2 = 0$

$\Rightarrow x = \frac{5 \pm \sqrt{17}}{2}$

$\boxed{1.2.7}$ (a) Obviously, $g(x)$ is continuous.

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And $g'(x) = \frac{1}{3}(2x-1)^{-2/3}$, i.e. g is differentiable.

Since $S = |g'(r)| = \frac{1}{3} < 1$,

By Theorem 1.6,

Fixed-point Iteration of $g(x)$ is locally convergent. □

(c) Obviously, $g(x)$ is continuous

And $g'(x) = \cos x + 1$, g is differentiable

Since $S = |g'(r)| = 2 > 1$,

By Theorem 1.6,

F-p Iteration ~~g(x) is~~ of $g(x)$ is NOT locally convergent. □

$\boxed{1.2.11}$

(a) (1) $x = x^3 + e^x$

(2) $x^3 = x - e^x$

$\Rightarrow x = (x - e^x)^{1/3}$

(3) $e^x = x - x^3$

$\Rightarrow x = \ln(x - x^3)$

$\boxed{1.4.1}$

(a). $g(x) = x^3 + x - 2$, $x_0 = 0$, $g'(x) = 3x^2 + 1$

$$x_1 = x_0 + \frac{g(x_0)}{g'(x_0)} = 2$$

$$x_2 = x_1 + \frac{g(x_1)}{g'(x_1)} = 18/13$$

(c) $f(x) = x^2 - x - 1$, $f'(x) = 2x - 1$, $x_0 = 0$

$$x_1 = x_0 + \frac{f(x_0)}{f'(x_0)} = -1$$

$$x_2 = x_1 + \frac{f(x_1)}{f'(x_1)} = -2/3.$$

$\boxed{1.4.3}$

(a) (1) $r = -1$

Since $f'(r) = 8 \neq 0$

By Theorem 1.11, $e_{i+1} \approx M e_i^2$, $M = \frac{f''(r)}{2f'(r)} = 5/2$

$\Rightarrow e_{i+1} \approx \frac{5}{2} e_i^2$

(2) $r = 0$

Since $f'(r) = -1 \neq 0$

By Theorem 1.11 $e_{i+1} \approx M e_i^2$

$\Rightarrow e_{i+1} \approx 2 e_i^2$

(3) $r=1$

Since $f'(r)=0$, the multiplicity at r is 3

$f''(r)=0$

$f'''(r)\neq 0$

By Theorem 1.12

$$e_{i+1}\approx S e_i,\quad S=\frac{m-1}{m},\quad m=3$$

$$\Rightarrow e_{i+1}\approx \tfrac{2}{3}e_i.$$

1.4.10 The fixed-point iteration is $x_{i+1}=g(x_i)$, where $g(x)=x-\dfrac{f(x)}{f'(x)}$

$$\Rightarrow x_{i+1}=x_i-\frac{x_i^3-A}{3x_i^2}$$