6.1.1 (b).

x	7	1	2
y	146	2	1

Totally, we have 3 points, thus we can find a second order polynomial to approximate this function.

By Lagrange polynomial,

$$P_2(x) = 146 \cdot \frac{(x-1)(x-2)}{(7-1)(7-2)} + 2 \cdot \frac{(x-7)(x-2)}{(1-7)(1-2)} + 1 \cdot \frac{(x-7)(x-1)}{(2-7)(2-1)}$$

6.1.9 Proof: To show $F(x) = g(x) + \frac{x_0 - x}{x_n - x_0}[g(x) - h(x)]$ interpolating $f(x)$ at $x_0, \cdots, x_n$,

we only need to show $F(x_i) = f(x_i)$, $i = 0, \cdots, n$.

Case (i): let $i = 0$, $F(x_0) = g(x_0) = f(x_0)$,

Case (ii): let $i = n$, $F(x_n) = h(x_n) = f(x_n)$.

Case (iii): Let $i = 1, \cdots, n-1$, $F(x_i) = g(x_i) = f(x_i)$

Therefore, $F(x_i) = f(x_i)$, $i = 0, \cdots, n$

$\square$

$\boxed{6.1.14}$ Proof. Apply Thm 2 on page 315

$$|p(x)-f(x)| \leq \frac{|f^{(n)}(\xi_x)|}{n!} \left|\prod_{i=0}^{n-1}(x-x_i)\right|$$

WLOG, let $x_0=0$, since $f^{(n)}(x)=\sinh(x)=\frac{e^x-e^{-x}}{2}$, n is even

$f^{(n)}(x)=\cosh(x)=\frac{e^x+e^{-x}}{2}$, n is odd

which implies $|f^{(n)}(x)| \leq 2$, $x \in [-1,1]$ ---(1)

Since $x_0=0$, $\left|\prod_{i=0}^{n-1}(x-x_i)\right| = \left|\prod_{i=1}^{n-1} x(x-x_i)\right| \leq |x| 2^{n-1}$

$x \in [-1,1]$ ---(2)

Use the fact that $|x| \leq |\sinh(x)|$ --- (3)

can be shown by looking at $g(x)=\frac{e^x-e^{-x}}{2}-x$

Combine (1), (2), and (3)

we have $|p(x)-f(x)| \leq \frac{2 \cdot 2^{n-1}}{n!}|x| \leq \frac{2^n}{n!}|f(x)|$.

$\boxed{6.1.22}$ (i) Lagrange form.

x	-2	0	1
$f(x)$	0	1	-1

$$P_2(x) = 0\cdot(\quad) + 1\cdot\frac{(x+2)(x-1)}{(0+2)(0-1)} - 1\cdot\frac{(x+2)(x)}{(1+2)(1)}$$

$$= -\frac{1}{2}(x^2+x-2) - \frac{1}{3}(x^2+2x)$$

$$= -\frac{5}{6}x^2 - \frac{7}{6}x + 1$$

(ii) Newton's form.

$$P_2(x) = C_0 + C_1(x+2) + C_2(x+2)(x-0)$$

Solve
$$\begin{cases} P(x_0) = C_0 = 0 \\ P(x_1) = C_0 + C_1(0+2) = 1 \\ P(x_2) = C_0 + C_1(1+2) + C_2(1+2)(1-0) = -1 \end{cases}$$
for C_0, C_1, C_2.

$$\Rightarrow \begin{cases} C_0 = 0 \\ C_1 = \frac{1}{2} \\ C_2 = -\frac{5}{6} \end{cases} \Rightarrow P_2(x) = 0 + \frac{1}{2}(x+2) - \frac{5}{6}(x+2)x$$

$$= -\frac{5}{6}x^2 - \frac{7}{6}x + 1$$

is identical as (i).

$\boxed{6.2.4}$ Apply Thm 4 on Page 333 (4)

Since f is a polynomial of degree k,

thus $f^{(n)}(\xi) = 0$, for $n > k$

$\Rightarrow f[x_0, x_1, \cdots, x_n] = \frac{1}{n!} f^{(n)}(\xi) = 0$.

$\boxed{6.2.8}$ Directly apply Thm 1 on Page 339

Lagrange form = Newton's form.

Both of them are the unique interpolation polynomial.

$\boxed{6.2.9}$ By 6.2.8, we have

Lagrange form = Newton's form.

which is to say the coefficents of x^n in

each form must be the same

$f[x_0, \cdots, x_n]$ is the coefficent of x^n term is Newton's form

at the same time the Lagrange form $P(x) = \sum_{i=0}^{n} f(x_i) \prod_{\substack{j \neq i \\ j=0}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$

has the coefficent for x^n: $\sum_{i=0}^{n} f(x_i) \prod_{\substack{j \neq i \\ j=0}}^{n} \frac{1}{(x_i - x_j)}$.

6.2.24

x	4	2	0	3
$f(x)$	63	11	7	28

$$p(x) = c_0 + c_1(x-4) + c_2(x-4)(x-2) + c_3(x-4)(x-2)x$$

	x	$f(x)$			
x_0	4	63	26	6	1
x_1	2	11	2	5	
x_2	0	7	7		
x_3	3	28			

where $\begin{cases} c_0 = 63 \\ c_1 = 26 \\ c_2 = 6 \\ c_3 = 1. \end{cases}$

6.3.1

x	0	1	2
$p(x)$	2	-4	44
$p'(x)$	-9	4	

Follow these ~~arrows~~ arrows in data. table, one has

$$p(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^2(x-1) + c_4 x^2(x-1)^2.$$

	x	$f(x)$				
x_0	0	2	$p'(0) = -9$	3	7	5
x_0	0	2	-6	10	17	
x_1	1	-4	$p'(1) = 4$	44		
x_1	1	-4	48			
x_2	2	44				

$\begin{cases} c_0 = 2 \\ c_1 = -9 \\ c_2 = 3 \\ c_3 = 7 \\ c_4 = 5 \end{cases}$

$\boxed{6.3.3}$ According to equations (8) (9) on page 343

For $p(x_i) = C_{i0}$ $p'(x_i) = C_{i1}$, $(0 \le i \le n)$

the Lagrange interpolation is given as

$$p(x) = \sum_{i=0}^{n} C_{i0} A_i(x) + \sum_{i=0}^{n} C_{i1} B_i(x)$$

where $A_i(x) = [1 - 2(x - x_i) l_i'(x_i)] l_i^2(x)$, $0 \le i \le n$

$B_i(x) = (x - x_i) l_i^2(x)$

Notice that Both of A_i and B_i are degree $2n+1$.

Hence, in our case $p(x_i) = y_i$, $p'(x_i) = 0$, $0 \le i \le n$.

the interpolation polynomial is

$$p(x) = \sum_{i=0}^{n} y_i A_i(x) + 0$$