HW4 MATH 170B (1)

$\boxed{6.4.5}$ According to the definition on page 349

To verify $f(x)$ is a 2nd order spline or not,

we only to to check $\begin{cases} S_0(1) = S_1(1) \\ S_1(2) = S_2(2) \end{cases}$, $\begin{cases} S_0'(1) = S_1'(1) \\ S_1'(2) = S_2'(2) \end{cases}$.

where $S_0(1) = 1 = S_1(1)$, $S_0'(1) = 0 = S_1'(1)$

$S_1(2) = \frac{3}{2} = S_2(2)$, $S_1'(2) = 0 = S_2'(2)$.

Hence, $f(x)$ is a quadratic spline.

$\boxed{6.4.6}$ No, $f(x)$ is not a cubic spline.

The reason is $S_0''(1) \neq S_1''(1)$.

$\boxed{6.4.7}$ Totally, we have 5 unknowns, then we need 5 eqs.

Inorder to be a cubic spline, we need $\begin{cases} S_0(1) = S_1(1) \\ S_1(3) = S_2(3) \end{cases}$, $\begin{cases} S_0'(1) = S_1'(1) \\ S_1'(3) = S_2'(3) \end{cases}$

and $\begin{cases} S_0''(1) = S_1''(1) \\ S_1''(3) = S_2''(3) \end{cases}$,

these give us $\begin{cases} a = c \\ c = d \end{cases}$

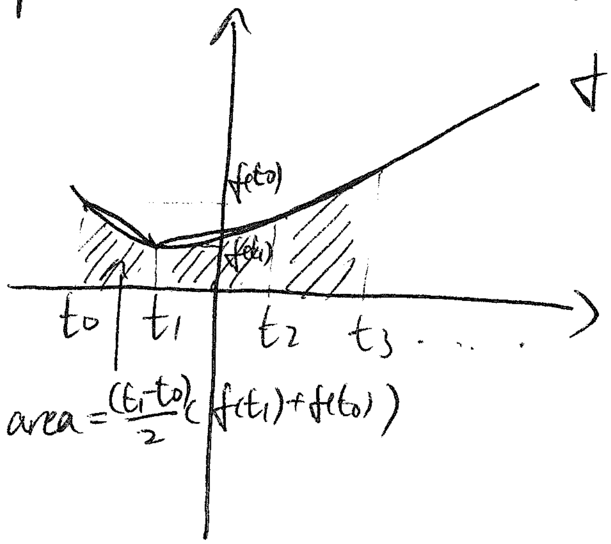
By the table, we can find $\begin{cases} 4a + b = 26 \\ c = 7 \\ 4d + e = 25 \end{cases}$

Solve $\begin{cases} a = c \\ c = d \\ 4a + b = 26 \\ c = 7 \\ 4d + e = 25 \end{cases} \Rightarrow \begin{cases} a = c = d = 7 \\ b = -2 \\ e = -3. \end{cases}$

$\boxed{6.4.24}$ For 1st degree spline, it is just the piece-wise ~~into~~ linear interpolation.

②

which is



then, the $\int_0^1 s(x)dx$ is the area of those trapezoidals.

Hence, the area can be calculated as

$$\int_0^1 s(x)dx = \sum_{i=0}^{n-1} \frac{(f(t_{i+1}) + f(t_i))(t_{i+1} - t_i)}{2}.$$

$\boxed{6.5.1}$ According to Eq (1) on Page 367.

$$B_j^2(t_i) = \left(\frac{t_i - t_j}{t_{j+2} - t_j}\right) B_j^1(t_i) + \left(\frac{t_{j+3} - t_i}{t_{j+3} - t_{j+1}}\right) B_{j+1}^1(t_i)$$

By the ~~$B_i(x)$~~ formula of $B_i^1(x)$ given on the bottom of page 367,

we have
$$B_j^1(x) = \begin{cases} 0, & x < t_j \text{ or } x \geq t_{j+2} \\ \dfrac{x - t_j}{t_{j+1} - t_j}, & t_j \leq x \leq t_{j+1} \\ \dfrac{t_{j+2} - x}{t_{j+2} - t_{j+1}}, & t_{j+1} \leq x < t_{j+2} \end{cases}$$

thus, $\cancel{B_j^2(t_i) \neq 0}$ only if $t_i = t_{j+1}$, then it equals

$\left(\frac{t_i - t_j}{t_{j+2} - t_j}\right) B_j^1(t_i) \neq 0$, to $\frac{t_{j+1} - t_j}{t_{j+2} - t_j} \cdot 1$

which is actually, $\left(\frac{t_i - t_{i-1}}{t_{i+1} - t_{i-1}}\right) \delta_{i,j+1}$

For the second term,

$\left(\frac{t_{j+3} - t_i}{t_{j+3} - t_{j+1}}\right) B_{j+1}^1(t_i) \neq 0$, only if $t_i = t_{j+2}$,

then, it equals to $\frac{t_{j+3} - t_{j+2}}{t_{j+3} - t_{j+1}} \cdot 1$

which is actually $\frac{t_{i+1} - t_i}{t_{i+1} - t_{i-1}} \cdot \delta_{i,j+2}$

$\boxed{6.5.2}$ By Lemma 1 on page 368, ④

that is if $k \geq 1$ and $x \notin (t_i, t_{i+k+1})$

then $B_i^k(x) = 0$.

Case I: When $k=0$, obviously $\sum_{i=-\infty}^{\infty} c_i B_i^k(x) = \sum_{i=m-k}^{m} c_i B_i^k(x)$

Case II: when $k \geq 1$, Since $x \in [t_m, t_{m+1})$, by Lemma 1,

only terms from $B_{m-k}^k(x)$ to $B_m^k(x)$ are non-zero.

thus, $\sum_{i=-\infty}^{\infty} c_i B_i^k(x) = \sum_{i=m-k}^{m} c_i B_i^k(x)$.

$\boxed{6.5.3}$ By 6.5.2, we have

$$S(t_j) = \sum_{i=-\infty}^{\infty} c_i B_i^2(t_j) = \sum_{i=j-2}^{j} c_i B_i^2(t_j)$$

By 6.5.1, we have $B_j^2(t_j) = 0$, and

$$S(t_j) = c_{j-2}\left(\frac{t_{j+1}-t_j}{t_{j+1}+t_{j-1}}\right) + c_{j-1}\left(\frac{t_j - t_{j-1}}{t_{j+1}+t_{j-1}}\right)$$

$$= c_{j-2}\frac{h_j}{h_j+h_{j-1}} + c_{j-1}\frac{h_{j-1}}{h_j+h_{j-1}}$$

$$= y_j$$

6.5.4 Show by Induction.

Step 1). Obviously, for $k=1$, $B_i^0(X) = B_0^0(X-t_i)$,

by the definition of $B_i^0(x)$.

Step 2). Assume $k=n$, $B_i^n(X) = B_0^n(X-t_i)$,

then, let's look at the case with $k=n+1$.

Simply apply eq (1) on page 367.

$$B_i^{n+1}(X) = \left(\frac{X-t_i}{t_{i+k}-t_i}\right) B_i^n(X) + \left(\frac{t_{i+k+1}-X}{t_{i+k+1}-t_{i+1}}\right) B_{i+1}^n(X).$$

By Assumption at $k=n$, we have

$$B_i^{n+1}(X) = \left(\frac{X-t_i}{t_{i+k}-t_i}\right) B_0^n(X-t_i) + \left(\frac{t_{i+k+1}-X}{t_{i+k+1}-t_{i+1}}\right) B_1^n(X-t_i)$$

$$= \left(\frac{(X-t_i)-t_0}{t_k-t_0}\right) B_0^n(X-t_i) + \left(\frac{t_{k+1}-X+t_i}{t_{k+1}-t_1}\right) B_1^n(X-t_i)$$

$$= B_0^{n+1}(X-t_i)$$

replace every k by n+1.

notice $t_i = i$.

$\boxed{6.6.1}$

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We just look at each ~~col~~ row.

Since $B_j^k(x_j) \neq 0$ for all j

$\Leftrightarrow$ By lemma 1, $t_j < x_j < t_{j+k+1}$

$$t_{j+1} < x_{j+1} < t_{j+k+2}$$

$$\vdots$$

$$t_{j+k} < x_{j+k} < t_{j+2k+1}$$

also $t_{j-1} < x_{j-1} < t_{j+k}$

$$\vdots$$

$$t_{j-k} < x_{j-k} < t_{j+1}$$

Notice that, all points from x_{j-k} to x_{j+k} may be in the interval (t_j, t_{j+k+1}).

this implies (again by lemma 1)

from $B_j^k(x_{j-k})$ to $B_j^k(x_{j+k})$ may be non-zero,

totally $2k+1$.