

Math 222 Final Review

1. Solve the following differential equation:

$$\frac{dy}{dx} = (4x - xy^2)^{3/2}$$

given the initial condition $y(0) = 0$ (You may assume $y(x) > 0$ for all x).

2. Calculate the Taylor Series for $f(x) = \frac{x}{1+x^4}$. Then find the values of x for which $T_\infty f(x) = f(x)$.

3. Compute the following integrals:

a. $\int \frac{x^2}{x^2 - x - 2} dx$

b. $\int \frac{1}{x(x-1)^2} dx$

4. For which values of p does the series

$$\sum_{n=1}^{\infty} \frac{(1+e^n)e^n}{e^{3np}}$$

converge?

5. Find the general solution to the following differential equations:

a. $\frac{dy}{dx} + (\cot x)y - \sin x = 0$

b. $\frac{dy}{dx} + y = \frac{\sin^3 x \cos^2 x}{e^x}$

6. What is an approximation to $\ln\left(\frac{3}{2}\right)$ within 0.02?

7. a. For $f(x) = e^{x^2} \sin x$, compute $f^{(5)}(0)$.

b. For $f(x) = \frac{1}{(1+x^3)^2}$, compute $f^{(27)}(0)$.

8. For which values of x does the series $\sum_{n=1}^{\infty} \frac{n^x e^n}{(2n)!}$ converge?

9. Let P be the plane containing the points $(0, 3, 1)$, $(-1, 1, 2)$, and $(0, 0, 0)$. Find the defining equation for P . Then find a parametric equation for the line normal to P which goes through the point $(2, 1, -1)$.

10. Find a reduction formula for $I_n = \int \frac{\sin x}{x^n} dx$ for $n \geq 3$. Show that $\int_0^\infty \frac{\sin x}{x^3} dx$ does not converge.

11. Let $f(x) = \int_0^x t^k e^{-t} dt$ for some $k \geq 0$. Compute $T_\infty f(x)$ and then show that $T_\infty f(x) = f(x)$ for all x .

12. Do the following series converge or diverge?

a. $\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{\sqrt{n}}$

b. $\sum_{n=1}^{\infty} \frac{n^4 + 6n - 3 + \cos(n)}{n^5 + 3n^3 - 2n + 1}$

13. Let $\vec{v} = (x, y, 2)$ and $\vec{w} = (2, 2, -1)$. Find two pairs of numbers (x, y) such that \vec{v} and \vec{w} are orthogonal and $\|\vec{v}\| = \|\vec{w}\|$.

Solutions

1. $y(x) = \frac{\frac{16}{5}x^{5/2}}{\sqrt{1 + \frac{64}{25}x^5}}$
2. $T_{\infty}f(x) = \sum_{n=0}^{\infty} (-1)^n x^{4n+1}; |x| < 1$
3. a. $x + \frac{4}{3} \ln|x-2| - \frac{1}{3} \ln|x+1| + c$
b. $\ln|x| - \ln|x-1| - \frac{1}{x-1} + c$
4. $p > \frac{2}{3}$
5. a. $y(x) = \frac{2x - \sin(2x) + c}{4 \sin x}$
b. $y(x) = \frac{1}{5}e^{-x} \cos^5 x - \frac{1}{3}e^{-x} \cos^3(x) + ce^{-x}$
6. $\frac{5}{12}$
7. a. 41
b. -10 (27!)
8. All x
9. $5x_1 - x_2 + 3x_3 = 0; (2, 1, -1) + t(5, -1, 3)$
10. $I_n = \frac{(1-n) \sin x}{x^{n-1}} - \frac{(n-1)(n-2) \cos x}{x^{n-2}} - (n-1)(n-2)I_{n-2}$
11. $T_{\infty}f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+k+1}}{n!(n+k+1)}$
12. a. Converge
b. Diverge
13. $(2, -1)$ and $(-1, 2)$