Math 222, Fall 2016 (Steven Sam)
Final review guide

List of topics to know for the final exam

(1) Methods of integration
  (a) Double-angle formulas
  (b) Integration by parts, reduction formulas
    (i) Repeated integration: Example 5.4, Example 6.2
    (ii) Taking $dv = 1$: Example 5.6
    (iii) Solving for integral: Example 5.7, Example 6.3
  (c) Partial fractions (only degree 2 and degree 3 denominators)
    (i) Long division
    (ii) Repeated roots, irreducible degree 2 polynomial
  (d) Trigonometric substitutions, completing the square

(2) Improper integrals
  (a) Two types of improperness (domain goes to $\pm\infty$ and function goes to $\pm\infty$), always split up integral so each one only has one form of improperness (improperness can be in middle of domain)
  (b) Key examples: p-test (Example 3.2) and exponential function (Example 3.1)
  (c) Convergence tests (remember to check hypotheses of tests)
    (i) Tail theorem
    (ii) Comparison test
    (iii) Limit comparison test

(3) Differential equations
  (a) Separable equations
  (b) First-order equations

(4) Taylor polynomials (from now on, always centered at $a = 0$)
  (a) If $m < n$, to get $T_m f(x)$ from $T_n f(x)$, delete all powers of x strictly bigger than m
  (b) If $p(x)$ is polynomial of degree d and $n \geq d$, then $T_n p(x) = p(x)$
  (c) Know examples in §3
  (d) Lagrange's formula for remainder
    (i) Know how to use this to bound remainder
    (ii) Approximating functions using Taylor polynomials with bound on error
  (e) Little-o notation
    (i) $R_n f(x)$ is $o(x^k)$ for $k \leq n$ (Theorem 8.2)
    (ii) Basic properties with addition and multiplication (p.85)
    (iii) Reading little-o from Taylor polynomials (Theorem 8.8)
  (f) Getting new Taylor polynomials from known ones
    (i) Addition: $T_n(f + g) = T_n f + T_n g$
    (ii) Substitution (Example 8.10)
    (iii) Multiplication (Example 8.11)
    (iv) Derivatives: $T_{n-1} f'(x) = (T_n f(x))'$ (Theorem 10.1)

(v) Antiderivatives: $T_{n+1}(\int_0^x f(t)dt) = \int_0^x (T_n f(t))dt$ (Example 10.3)

(5) Sequences and series
  (a) Limits of sequences
    (i) Definition 2.2 (I won't test using this formal definition, but you should intuitively understand what it means)
    (ii) Limit of powers of a number (Example 2.4)
    (iii) Basic laws (Theorem 2.5)
    (iv) Sandwich theorem (Theorem 2.6)
    (v) Applying functions (Theorem 2.7)
    (vi) If $\lim_{x\to\infty} f(x)$ exists and $a_k = f(k)$, then $\lim_{k\to\infty} a_k = \lim_{x\to\infty} f(x)$.
    (vii) Factorial beats exponential (Example 2.11)
  (b) Convergence of series
    (i) Definition 4.1: it is limit of partial sums
    (ii) Geometric series (Example 4.2)
    (iii) Basic laws (Theorem 4.4)
    (iv) If $\lim_{n\to\infty} |a_n|$ is not 0, then $\sum_{k=1}^{\infty} a_k$ diverges.
    (v) Convergence tests from handout (alternating, integral, comparison, limit comparison, ratio)
  (c) Convergence of Taylor series
    (i) To check where Taylor series converges, can use ratio test, then test endpoints using alternating series or something else. See Example 6 from handout.
    (ii) To check if $T_\infty f(x) = f(x)$, need to show that $\lim_{n\to\infty} |R_n f(x)| = 0$. Good examples to study:
      (A) $\frac{1}{1-x}$ (Example 5.1)
      (B) e^x (Example 5.2)
      (C) $\sin x$ (Example 7 from handout)
      (D) $\ln(1+x)$ (Section 5.7)
    (iii) $R_n f(x)$ compatible with addition, derivatives, substitutions, antiderivatives, just like $T_n f(x)$. Multiplication is more subtle, but you have $T_\infty(fg) = (T_\infty f)(T_\infty g)$.

(6) Vectors
  (a) Vector algebra (adding, scalar multiplication, length, etc.) and basic laws (§6.1.5)
  (b) Geometric interpretation of vectors
  (c) Parametric equations for lines
  (d) Dot product
    (i) Basic laws (§6.5.2)
    (ii) Using normal vector to get equation of lines and planes
  (e) Cross product
    (i) Basic laws (§6.6.5)
    (ii) Finding normal of plane

(7) Miscellaneous
  (a) convergent + convergent = convergent

(b) divergent + convergent = divergent

(c) For $a \leq b$, $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$.

Things in the book, but not on the final

- Rational substitutions
- §§3.7–3.8: Direction fields, Euler's method
- §3.10: Differential equations word problems
- Binomial formula (in §4.3)
- Fibonacci numbers (§§4.8.12–4.8.13)
- Proofs from chapter 4 (§§4.12–4.13)
- Example 5.5.13
- §6.4: Vector bases
- Relation of dot product to angles (Theorem 5.7)
- Theorem 6.5.5
- Orthogonal projection (§6.5.8)
- Distance to line (§6.5.11)
- Triple products, determinants
- Area of parallelogram