Math 222 (Steven Sam), Fall 2016
Homework 4, due October 5

Only the starred problems (7 total) need to be submitted for grading.

Chapter 2.4 (pages 47–48) from book: 14, 16
Chapter 2.6 (pages 55–56) from book: 2, 4, 5, 9*, 11, 12, 13*, 15, 16*

(E1)* Evaluate $\int_0^3 \frac{dx}{x-1}$.

(E2) Evaluate $\int_1^\infty \frac{dx}{x^2-4}$.

(E3) Show that $\int_1^\infty \frac{1+e^{-x}}{x}\,dx$ is divergent.

(E4)* Determine all p such that $\int_2^\infty \frac{dx}{x(\ln x)^p}$ converges and calculate its value when it does converge.

(E5) Determine all p such that $\int_1^\infty \frac{\ln x}{x^p}dx$ converges and calculate its value when it does converge.

(E6)* Recall from Math 221 that if $y = f(x)$ is the graph of a function, then the volume of the solid of revolution (from a to b) around the x-axis is given by

$$V = \pi \int_a^b f(x)^2\,dx.$$

Its surface area is given by

$$A = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx.$$

Take $f(x) = \frac{1}{x}$, $a = 1$, and $b = \infty$.

(a) Sketch the graph and imagine what the solid of revolution looks like.
(b) Show that V is finite and compute it.
(c) Show that A is infinite.

So you get a curious shape that “can’t hold enough paint to paint itself”.

(E7)* Let $x^3 + cx^2 + dx + e$ be a polynomial which is nonzero if $x \geq A$.

(a) Use the comparison test to show that $\int_A^\infty \frac{x^2+ax+b}{x^3+cx^2+dx+e}dx$ diverges for all choices of coefficients a, b.

(b) Use the comparison test to show that $\int_A^\infty \frac{x+a}{x^3+cx^2+dx+e}dx$ converges for all choices of a.