List of topics for Math 184A final exam

The final exam is 3 hours long and it will be roughly the length of 2 midterms or shorter. Just like the midterms, the style of problems will be similar to past homework. There is no need to memorize proofs from class. However, the more of them that you understand, the more it will help to solve problems. Here is a rough outline of the things to expect (not meant to be detailed, but enough to remind you of the topics we discussed):

1. Basic counting problems

(1) 12-fold way: understand all entries, especially ordered vs. unordered
(2) Proof techniques: induction, bijections, recursive formulas
(3) Important examples
  (a) Counting subsets (all, of specific size, Pascal's identity, bijection with words)
  (b) Multisets (and their many interpretations: compositions, monomials, solutions to equations like HW2#1; bijection with subsets, Example 3.15, proof of Theorem 3.16)
  (c) Permutations
  (d) Ordering elements with duplicates (flower problem, Example 3.6 or rearranging letters of a word)
  (e) Set partitions (recursive formula, connection to surjective functions)
  (f) Integer partitions (definitions, Young diagrams, transpose operation)

2. Useful formulas

(1) Binomial theorem: modify to get new identities using derivatives, integration, substitution, taking products (see solution of HW4#4)
(2) Inclusion-exclusion
  (a) Understand statement for arbitrary number of sets
  (b) Examples: derangements, Stirling numbers, number of integers with certain properties (HW4#1), lists without consecutive repetitions

3. Generating functions

(1) Formal power series and algebraic manipulations
  (a) Adding, multiplying, compositions, derivatives, re-indexing
  (b) General binomial theorem
  (c) Writing as rational functions (when possible)
  (d) Extracting formulas for coefficients (examples in notes and HW)
(2) Ordinary generating functions
  (a) Solving linear recurrence relations
  (b) Product and composition formulas (best used when dealing with *ordered* sets)
  (c) Product representations for OGF of partition functions
  (d) Formulating and/or solving non-linear recurrence relations (e.g., Catalan numbers). There isn't really a general formula here, but there is a general approach.
(3) Exponential generating functions
  (a) Product and composition formulas (best used when dealing with *unordered* sets)
  (b) Examples: set partitions (possibly with conditions on block sizes or number of blocks)

(4) Lagrange inversion
  (a) Using to find coefficients of formal power series
  (b) Sometimes not in the form $A(x) = xG(A(x))$, so might need to rearrange terms or define new power series (see examples in HW and notes)

4. Partially ordered sets

(1) Computing Möbius functions for general poset
(2) Knowing Möbius function for Boolean poset and divisor poset
(3) Counting necklaces example

5. Extra practice from Bona (3rd edition)

Some of these problems require partial fraction decomposition to completely solve. However, I won't test you on that.

All of these have solutions in the book.

- Chapter 3: 1-3, 6-12, 14-15, 18-20, 23
- Chapter 4: 3-5, 7-9, 17-18, 25-27
- Chapter 5: 1, 4-8, 11-13, 16
- Chapter 7: 3-11, 13
- Chapter 8: 1-10, 12-17, 22 (combinatorial proof means direct reasoning, without recursions or generating functions)

The following do not have solutions in the book. I will not provide a solutions manual due to time constraints. However, I am happy to discuss these problems either in office hours or over Piazza.

- Chapter 3: 25-35, 37-38, 40-46, 48-51
- Chapter 4: 31-32, 35-36, 38-40, 45-46, 49
- Chapter 5: 17-28, 32-33, 35-36
- Chapter 7: 15, 17, 20, 23-27, 36
- Chapter 8: 23-25, 27-29, 34-36, 39, 43-45, 49

6. Extra practice from homework assignments

Some of the problems in homework can be modified to create new problems. For maximum effect, write your own variations of these problems and exchange with a classmate. Here are some suggestions:

- HW2
  - #1: Change number of variables; change 96 to another number; put conditions on what values x_i can take.
  - #2: Choose your favorite word
  - #3: Change numbers
  - #4, #6: Make up your own conditions on how the subsets interact
- HW3
  - #1: Change 56; what happens if even is replaced with odd? or divisible by 3? etc.
  - #2: Find formulas for $S(n, n-3)$, $S(n, n-4)$, etc. or $S(n, 3)$, $S(n, 4)$, etc.
- HW4
  - #1: Change 1000; change conditions on numbers or add conditions

  - #3: Change number of letters that repeat
  - #4, #5: There are plenty of "binomial identities" both in the book and online. Try to look some up and see if you can prove them yourself. Also, see if you can prove them in different ways (induction, binomial theorem, bijective proof)
- HW5
  - #1, #2: Change numbers but keep general structure of recursions the same. Can we add more terms and solve the problems using the same method?
  - #3: Put in more complicated rational functions. What happens if the denominator has two different factors? The more factors in the denominator, the more computationally messy it will be.
- HW6
  - #1: Change numbers
  - #2: This one is about compositions of OGF. Try to make up your own problems with the same structure and exchange with someone else.
  - #3: These kinds of problems are fairly limited and hard to find good examples of. However, it is good practice to write down product formulas for OGF of partition functions with different conditions on which numbers are allowed and how many times they may appear.
- HW7
  - #1: Add more colors or put your own conditions on how many are allowed of each color.
  - #2: Similar to advice for HW6 #2
  - #4: Put different conditions on block sizes that are allowed. Advice is similar to HW6#3.
  - #5: What if we want the 4-fold composition $f \circ f \circ f \circ f$ to be the identity? Or 5-fold? etc.
- HW8
  - #2, #3: Use other posets
  - #4: Different values of n