Math 376, Spring 2018
Homework 3
Due: February 14, 2018 in your discussion section

(1) For each vector field $f$ defined on $U \subseteq \mathbf{R}^n$, determine if $f$ is a gradient. If yes, construct a potential function for $f$.
  (a) $U = \mathbf{R}^3$ and $f(x, y, z) = (y^3, 3xy^2 + e^{3z}, 3ye^{3z})$.
  (b) $U = \mathbf{R}^2$ and $f(x, y) = (\sin x, \cos y)$.
  (c) $U = \mathbf{R}^3$, $f(x, y, z) = (x + z, -y - z, x - y)$.
  (d) $U = \mathbf{R}^3 \setminus \{(0, 0, 0)\}$ and
$$f(x, y, z) = \left( \frac{x}{(x^2 + y^2 + z^2)^2}, \frac{y}{(x^2 + y^2 + z^2)^2}, \frac{z}{(x^2 + y^2 + z^2)^2} \right).$$
  (e) $U = \mathbf{R}^3$ and $f(x, y, z) = (xy, \frac{y^2}{2}, yz)$.

(2) Let $Q = [a, b] \times [c, d]$ be a rectangle and let $f\colon Q \to \mathbf{R}$ be a step function. Show that the value of $\iint_Q f$ does not depend on the choice of subdivision of $Q$ as long as $f$ is constant on each rectangle.

(3) Prove Theorem 3.2 from the notes.

(4) Let $f\colon [0, 1] \to \mathbf{R}$ be the function defined by
$$f(x) = \begin{cases} 1 & \text{if } x \text{ is a rational number} \\ 0 & \text{if } x \text{ is irrational} \end{cases}.$$
Compute the upper and lower integrals of $f$. Is $f$ integrable?