Math 376, Spring 2018
Homework 4
Due: February 21, 2018 in your discussion section

(1) Integrate the following functions on $Q = [0, 1] \times [0, 1]$.
  (a) $f(x, y) = xy$.
  (b) $f(x, y) = e^x y + xe^y$.
  (c) $f(x, y) = x \sin(xy)$.

(2) (Apostol 11.9.14) Define f on $Q = [0, 1] \times [0, 1]$ by

$$f(x, y) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{if } x \neq y \end{cases}.$$

Prove that f is integrable and show that $\iint_Q f = 0$.

(3) (Apostol 11.15.8)

(4) Let Q be a rectangle and let f, g be integrable functions on Q.
  (a) Given real numbers c_1, c_2, show that

$$\iint_Q (c_1 f + c_2 g) = c_1 \iint_Q f + c_2 \iint_Q g.$$

  (b) If Q is broken up into two rectangles Q_1 and Q_2, show that

$$\iint_Q f = \iint_{Q_1} f + \iint_{Q_2} f.$$

  (c) If $f(x) \geq g(x)$ for all $x \in Q$, show that

$$\iint_Q f \geq \iint_Q g.$$

(5) Prove the properties about bounded sets of content 0 from Apostol, §11.11:
  (a) A finite set of points in the plane has content 0.
  (b) A union of a finite number of bounded sets of content 0 is also of content 0.
  (c) If S has content 0 and $S' \subseteq S$, then S' also has content 0.
  (d) A line segment has content 0.