Math 376, Spring 2018
Homework 6
Due: March 14, 2018 in your discussion section

(1) Let E be the region in $\mathbf{R}^3$ described by $x^2 + y^2 \leq 1$ and $1 - x^2 - y^2 \leq z \leq 1$. Let $f(x, y, z)$ be a continuous function on E. Setup the integral $\iint_E f(x, y, z)$ as an iterated integral in cylindrical coordinates.

(2) Use spherical coordinates to compute the volume of the region in $\mathbf{R}^3$ bounded by the sphere $z = x^2 + y^2 + z^2$ and the cone $z = \sqrt{x^2 + y^2}$. [This looks like an ice cream cone.]

(3) Evaluate $\displaystyle\iint_{\mathbf{R}^3} \frac{1}{(x^2 + y^2 + z^2 + 1)^2}$.

(4) Prove the following properties about cross products:
- $\mathbf{a} \times \mathbf{a} = \mathbf{0}$
- $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
- $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
- $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$
- $\mathbf{a} \times \mathbf{b}$ is orthogonal to $\mathbf{a}$ and $\mathbf{b}$, i.e., $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = 0$ and $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = 0$

[All of these can be deduced from properties of determinants rather than expanding out the expressions – I recommend trying to do it that way.]