Math 184A, Midterm 1
Instructor: Steven Sam
October 24, 2018

Name:

Also write your name on the back of the last page.

Problem	Score
1	/ 12
2	/ 10
3	/ 10
4	/ 8
Total	/ 40

- No books, materials, notes, cell phones, calculators, etc. are allowed during the exam.

- Cross out or erase irrelevant scratch work. If you write incorrect statements without crossing them out, you may lose points. **It must be clear what your final answer is.**

- When asked to explain or prove, give enough detail so that we know that you are not guessing the answer. We are not mind readers, so you will not receive the benefit of the doubt if you skip too much detail.

- If you need more space, you may use the backs of the pages and also there is a blank sheet at the end. Please clearly indicate which problem you are working on. If you still need more paper, raise your hand.

Good luck!

1. (4+5+3 points) You don't need to explain your answer, but a wrong answer with no explanation might receive 0 points.

   (a) List all of the integer partitions of 5.

   (b) How many set partitions of [4] are there? Give your answer as a number.

   (c) How many 3-element subsets of [6] are there? Give your answer as a number.

2. (5+5 points) Give a brief explanation of your answers. You don't need to simplify the final answer.

(a) How many ways are there to list the letters of the word SASSAFRAS?

(b) How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 40$$

where each x_i is a non-negative integer, and $x_6 \neq 0$?

3. (6+4 points) Let n, k be positive integers. Consider the set of sequences $(X_1, \ldots, X_k)$, where each X_i is a subset of $[n]$ and $X_1 \subseteq X_2 \subseteq \cdots \subseteq X_k$ (i.e., X_i is a subset of X_{i+1} for $i = 1, \ldots, k-1$). Give a brief explanation of your answers to:

(a) How many sequences are there?

(b) Pick i such that $1 \le i \le k$. How many sequences satisfy $X_i \neq \emptyset$?

4. (8 points) Let $A(n)$ be the number of compositions of n such that all parts are odd. For $n \geq 3$, prove that
$$A(n) = A(n-1) + A(n-2).$$

Space for extra scratch work