Math 184A, Midterm 2
Instructor: Steven Sam
November 19, 2018

Name:

Discussion time (circle): 4PM 5PM 7PM 8PM

Also write your name on the back of the last page.

Problem	Score
1	/ 12
2	/ 10
3	/ 10
4	/ 8
Total	/ 40

- No books, materials, notes, cell phones, calculators, etc. are allowed during the exam.
- Cross out or erase irrelevant scratch work. If you write incorrect statements without crossing them out, you may lose points. **It must be clear what your final answer is.**
- When asked to explain or prove, give enough detail so that we know that you are not guessing the answer. We are not mind readers, so you will not receive the benefit of the doubt if you skip too much detail.
- If you need more space, you may use the backs of the pages and also there is a blank sheet at the end. Please clearly indicate which problem you are working on. If you still need more paper, raise your hand.

Good luck!

1. (4+4+4 points) You don't need to explain your answer, but a wrong answer with no explanation might receive 0 points.

   (a) Simplify $\sum_{i=0}^{n} i \binom{n}{i} 4^{i-1}$.

   (b) Express $\sum_{n \geq 0} (2^n - 3n)x^n$ as a rational function.

   (c) Give the formula for the Catalan number C_n.

2. (5+5 points) Show your work.

(a) Define a sequence of integers $a_0, a_1, \ldots$ by the conditions

$$\begin{aligned} a_0 &= 3, \qquad a_1 = 7, \\ a_n &= 5a_{n-1} - 6a_{n-2} \qquad \text{for } n \geq 2. \end{aligned}$$

Give a closed formula for a_n.

(b) If $\displaystyle\sum_{n\geq 0} b_n x^n = \frac{1+x^2}{(1-3x)^5}$, give a closed formula for b_n.

3. (4 + 6 points) Give a brief explanation of your answers.

   (a) How many ways are there to list the letters of the word LAJOLLA so that there are at least 2 consecutive L's?

   (b) How many ways are there to list the letters of the word LAJOLLA so that no two consecutive letters are the same?

4. (8 points) Let a_n be the number of partitions of n such that all of the parts are odd, and each number appears ≤ 3 times. By convention, $a_0 = 1$.

   For example, for $n = 6$, we get the partitions $\{(5,1), (3,3), (3,1,1,1)\}$, so $a_6 = 3$.

   Let b_n be the number of partitions of n such that every part is different, and none of them is divisible by 4. By convention, $b_0 = 1$.

   For example, for $n = 6$, we get the partitions $\{(6), (5,1), (3,2,1)\}$, so $b_6 = 3$.

   Prove that $a_n = b_n$ for all n.

Space for extra scratch work