Math 200C, Spring 2022
Homework 5
Due: May 27 11:59PM via Gradescope

Please do not search for solutions. I would rather help you directly (via office hours or Discord) so that I can calibrate explanations in the notes and lecture. You are free to work with other students, but solutions must be written in your own words. Please cite any sources (beyond the course materials) that you use or any people you collaborated with.

This covers the material up through Section 6 of the notes (lectures 14–17).

(1) Atiyah–Macdonald, Exercise 11.4. (There isn't much to solve here, so just put the pieces together and solve Exercise 7.9.)
(2) Atiyah–Macdonald, Exercise 11.6
(3) Atiyah–Macdonald, Exercise 11.7. Feel free to use Exercise 3.21 without proof.
(4) Let $\mathbf{k}$ be a field and let A and B be finitely generated $\mathbf{k}$-algebras. Show that

$$\dim(A \otimes_{\mathbf{k}} B) = \dim A + \dim B.$$

1. Extra problems (don't submit)

(5) Let $\mathbf{k}$ be a field and let A and B be finitely generated $\mathbf{Z}_{\geq 0}$-graded $\mathbf{k}$-algebras with $A_0 = B_0 = \mathbf{k}$. The **Segre product** of A and B is the subalgebra

$$A \boxtimes B = \bigoplus_{d \geq 0} A_d \otimes_{\mathbf{k}} B_d$$

of $A \otimes B$. Show that $A \boxtimes B$ is a finitely generated $\mathbf{k}$-algebra and compute its dimension in terms of $\dim A$ and $\dim B$.