Math 184, Midterm 1
Instructor: Steven Sam
October 23, 2019

Name and PID:

Discussion time (circle): 4PM 5PM

Also write your name on the back of the last page.

Problem	Score
1	/ 12
2	/ 12
3	/ 8
4	/ 8
Total	/ 40

- No books, materials, notes, cell phones, calculators, etc. are allowed during the exam.
- Cross out or erase irrelevant scratch work. If you write incorrect statements without crossing them out, you may lose points. **It must be clear what your final answer is.**
- When asked to explain or prove, give enough detail so that we know that you are not guessing the answer. We are not mind readers, so you will not receive the benefit of the doubt if you skip too much detail.
- If you need more space, you may use the backs of the pages and also there is a blank sheet at the end. Please clearly indicate which problem you are working on. If you still need more paper, raise your hand.

Good luck!

1. (4+4+4 points) You don't need to explain your answer, but a wrong answer with no explanation might receive 0 points. You don't need to simplify answers.

   (a) What is the value of $S(9, 2)$ (Stirling number)?

   (b) How many positive integer solutions are there to the equation $a + b + c + d = 50$?

   (c) How many ways are there to rearrange the letters of the word BOOKKEEPER?

2. (6+6 points) Give a brief explanation of your answers. You don't need to simplify.

   (a) Given 12 cats and 10 dogs, how many ways are there to choose a group of 6 of them if we require that the group has at least 1 cat?

   (b) You choose 6 cards from a standard deck of cards (4 suits, 13 values). How many ways are there to get 3 pairs? i.e., 2 of the cards have the same value, another 2 have the same value, and the last 2 also have the same value (and all 3 of these values are different, i.e., there is no 4 of a kind).

3. (8 points) How many ways can we pick subsets A, B, C of $[n]$ so that $A \cup B \cup C = [n]$ and $A \cap B \cap C \neq \emptyset$?

4. (8 points) Let $A(n)$ be the set of subsets $S \subseteq [n]$ with no consecutive elements, i.e., if $i \in S$ then $i+1 \notin S$. For $n \geq 3$, prove that

$$|A(n)| = |A(n-1)| + |A(n-2)|.$$

Space for extra scratch work