Definition of a vector space over \( \mathbb{R} \): A vector space (over \( \mathbb{R} \)) is a nonempty set \( V \) of objects called vectors. The vector space comes with two operations on its vectors:
- addition, denoted +
- scalar multiplication

The operations must satisfy the following ten axioms for every \( \vec{u}, \vec{v}, \vec{w} \in V \) and every \( c, k \in \mathbb{R} \):
1. There is a “zero vector”, denoted \( \vec{0} \), in \( V \) with \( \vec{0} + \vec{v} = \vec{v} \).
2. \( \vec{u} + \vec{v} \in V \)
   **Language note:** We say that “\( V \) is closed under addition” because when you add two vectors in \( V \), their sum does not escape from \( V \).
3. \( c\vec{v} \in V \)
   **Language note:** We say that “\( V \) is closed under scalar multiplication” because when you multiply a vector in \( V \) by a scalar, the result does not escape from \( V \).
4. \( \vec{u} + \vec{v} = \vec{v} + \vec{u} \)
5. \( \vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w} \)
6. If \( \vec{v}_1 \in V \), then there is a \( \vec{v}_2 \in V \) with \( \vec{v}_1 + \vec{v}_2 = \vec{0} \).
7. \( (c + k)\vec{v} = c\vec{v} + k\vec{v} \)
8. \( (ck)\vec{v} = c(k\vec{v}) \)
9. \( c(\vec{v} + \vec{w}) = c\vec{v} + c\vec{w} \)
10. \( 1\vec{v} = \vec{v} \)