Name: _______________________________ PID: ________________

TA: __________________ Sec. No: _____ Sec. Time: _____

Math 20B.
Midterm Exam 1
January 27, 2011

Turn off and put away your cell phone.
You are not allowed to use a calculator during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

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1. (9 points) Evaluate the given expression:

(a) \( \frac{d}{dx} \left( \int_{\frac{e}{2}}^{e} \left( \frac{2t}{1 + t^3} \right)^{1/4} dt \right) \)

(b) \( \int_{0}^{\pi/2} \sin^6 x \cos x \, dx \)
(c) \[ \int \frac{dx}{x \ln(\sqrt{x})} \]
2. (6 points) A particle is moving along a straight line with velocity \( v(t) = 6t^2 + 6t - 12 \) feet per second.

(a) Find the average velocity of the particle (i.e. the average value of the function \( v \)) on the interval \([0, 2]\).

(b) Find the distance travelled by the particle on the interval \([0, 2]\).
3. (9 points) Let $\mathcal{R}$ be the region enclosed by the curves $y = x^2 + 2$, $y = 0$, $x = 0$ and $x = 2$, as shown in the figure to the right.

(a) Write down (but do not evaluate) an expression involving definite integrals that equals the volume of the solid obtained by rotating $\mathcal{R}$ about the $x$-axis.

(b) Write down (but do not evaluate) an expression involving definite integrals that equals the volume of the solid obtained by rotating $\mathcal{R}$ about the $y$-axis.

(c) Write down (but do not evaluate) an expression involving definite integrals that equals the volume whose base is $\mathcal{R}$ and whose cross-sections perpendicular to the $x$-axis are semi-circles.
4. (3 points) Write down (but do not evaluate) an expression involving definite integrals that equals the area of the region that lies inside the curve $r = 3 + 2 \sin \theta$ and outside the circle $r = 2$. 

Polar region $2 < r < 3 + 2 \sin(\theta)$