Mathematics 102: "Applied Linear Algebra" Syllabus

Lecture schedule based on:
Linear Algebra, fourth edition by Stephen H. Friedberg, Arnold J. Insel, and Lawrence E. Spence

Lecture	**Section(s)**	**Topic(s)**
0	3.1 - 3.4	elementary matrix operations; Gaussian elimination; (reduced) row echelon form; rank; elementary matrices; matrix inverse; linear systems of equations
1	1.2 - 3.4	formal definition of vector space and examples; subspaces and examples; linear combinations and examples
2	1.5 - 1.6	linear (in)dependence; bases; the Replacement (aka Exchange) Theorem (1.10); dimension; dimension of subspaces
3	2.1 - 2.2	linear transformations; image and nullspace; the Dimension Theorem (2.3); matrix representations
4	2.3 - 2.5	matrix multiplication vs. function composition; invertibility and isomorphism; change of basis matrix
5	4.1 - 4.2	2×2 determinants; oriented area of parallelograms; larger determinants and examples
6	4.2 - 4.3	determinant as a multilinear function; cofactor expansion; relationship to rank; multiplicativity
7	4.5*	(* this topic may be skipped) characterization of $\det_n$ as the only normalized alternating multilinear functional on $M_{n\times n}$
8	5.1	eigenvectors and eigenvalues; characteristic polynomials; examples
9	5.2	linear independence of eigenvectors; diagonalizability; multiplicity; eigenspaces; examples (from ODE)
10	5.2, 5.4	direct sums $\oplus$; invariant subspaces; the Cayley-Hamilton Theorem
11	6.1	norms and inner products; orthogonality; examples
12	6.2	orthogonal projections and orthogonal complements; the Gram-Schmidt orthogonalization process
13	6.3	adjoints of linear operators (and relation to conjugate transpose); least squares approximation
14	6.4	Schur's Theorem (6.4); normal operators; self-adjoint and skew self-adjoint operators
15	6.5	orthogonal/unitary operators; unitary equivalence; conic sections
16	6.6	characterization of orthogonal projections ($T = T^* T$); the Spectral Theorem (6.25)
17	6.7	singular values; the Singular Value Decomposition (SVD)
18	6.7	polar decomposition; Moore-Penrose pseudoinverse
19	6.8	quadratric forms and matrix representations; congruence; symmetric bilinear forms
20	6.8	diagonalization of quadratic forms; Sylvester's "Law of Inertia"; application to the second derivative test in multivariate calculus
21	6.11*	(* this topic may be skipped) decomposition of orthogonal operators into reflections and rotations
22	7.1	non-diagonalizable matrices; generalized eigenvalues and eigenspaces
23	7.1 - 7.2	cycles of generalized eigenvectors; Jordan bases; Jordan blocks; Jordan Canonical Form
24	7.2	uniqueness of Jordan Canonical Form; *lots* of examples
25	7.3*	(* this topic may be skipped) the minimal polynomial

Notes:

1. This syllabus is designed for a 1-quarter course with 30 academic hours of instruction. It is sectioned into 25 lectures; this leaves 2 lectures available for in-class midterm exams and 3 lectures for review (and slack). It is based on the following textbook:
   - *Linear Algebra, fourth edition* by Stephen H. Friedberg, Arnold J. Insel, and Lawrence E. Spence
2. This course has Math 18 (previously numbered Math 20F) and Math 20C as a *prerequisite*. Thus, the students are expected to have a working knowledge of linear algebra and multivariable calculus.
3. Math 18/20F amply covers the content of Chapters 1-5 in this textbook in full rigorous detail (but perhaps at a slightly lower level of mathematical maturity). *This course will not focus on reviewing that material.* While the beginning of the course will spend some time repeating some of these topics at a higher level of sophistication (particularly Chapters 4 and 5), the students are expected to know this material already. If any students claim they have never seen this material, *do not* reformulate the course to re-teach it; those students should be instructed that they are not ready to take Math 102, and should review their Math 18/20F course before proceeding.
4. There is a listed "Lecture 0." This material (from Chapter 3, on the more computational aspects of matrix operations, and thoroughly covered in the prerequisite course) is not to be covered in lecture at all; rather, the instructor should create a (long) homework assignment based on this material to assign on the first lecture (but not necessarily to be turned in or graded). The point of "Lecture 0" is to announce the prerequisite material for the course.
5. Lectures 1-4 review material from Chapters 1-2 at a very fast pace, essentially just to remind the students of the important definitions and theorems going forward.
6. There are 3 topics (in Lectures 7, 21, and 25) that may be skipped in the event that the instructor still needs more time; however, these should be included if at all possible.