Based on *Vector Calculus* (6th edition) by Marsden and Tromba.
[See the next pages for explanatory comments.]

Lecture	Section	Topic
1	1.1	Vectors in Two- and Three-Dimensional Space
2–3	1.2	The Inner Product, Length, and Distance
4–5	1.3	Matrices, Determinants, and the Cross Product
6	2.1	The Geometry of Real-Valued Functions
7	2.2	Limits and Continuity
8–9	2.3	Differentiation
10	2.4	Introduction to Paths and Curves
11–12	2.5	Properties of the Derivative
13–14	2.6	Gradients and Directional Derivatives
15	3.1	Iterated Partial Derivatives
16–17	3.3	Extrema of Real-Valued Functions
18–19	3.4	Constrained Extrema and Lagrange Multipliers
20	4.1	Acceleration and Newton's Second Law
21	4.2	Arc Length
22	5.1	Introduction (to Double and Triple Integrals)
23	5.2	The Double Integral over a Rectangle
24	5.3	The Double Integral over More General Regions
25	5.4	Changing the Order of Integration
26*	5.5	The Triple Integral

*This topic may be omitted.

Math 20C is the third quarter course in calculus for students majoring in Mathematics, Engineering and the sciences. Most students taking Math 20C will be continuing from Math 20B, but some new freshmen are placed directly into Math 20C with Advanced Placement credit. This makes teaching Math 20C in a Fall quarter interesting: it will have both incoming freshmen with Advanced Placement credit and continuing sophomores who failed (or did not take) Math 20C the previous Spring quarter.

Math 20C introduces vectors and three-dimensional geometry and covers multivariable differential calculus with an introduction to multiple integrals. Experience has shown that students have more trouble visualizing the geometry of space and understanding the geometrical significance of the calculus than they do with the actual computations. Thus, more emphasis should be placed on *what* is being computed and *why* it is being computed than on how to compute it.

In this textbook, topics are sometimes treated in greater generality than strictly necessary for Math 20C. Instead of considering functions from n-dimensional space to m-dimensional space, emphasis should be placed on real-valued functions and curves in two- or three-dimensional space.

The following syllabus requires 26 lectures of the 28 to 30 lectures available in a typical quarter. Some topics can expanded if time permits.

Remarks about Topics

Lec. 1. Sec. 1.1: Vectors in two and three dimensions: Parametric equations for a line are introduced in the context of three-dimensional vectors.

Lec. 2–3. Sec. 1.2: The inner product (or dot product). Include orthogonal projections.

Lec. 4–5. Sec. 1.3: The cross product. There is not enough time for a full treatment of determinants, so show how to compute three-by-three determinants, but do not go through all of the properties. Focus instead on computing cross products. Include the equation of a plane, but skip "Distance: Point to a Plane".

Section 1.4 is covered in Math 20E.

Lec. 6. Sec. 2.1: Graphs of functions. Level curves and surfaces. Emphasis should be placed on graphs in two- and three-dimensional space.

Lec. 7. Sec. 2.2: Limits and continuity for real-valued functions. This discussion should be kept informal. The epsilon-delta definition was not covered in Math 20A, and so is not necessary here. Aim for intuitive understanding.

Lec. 8–9. Sec. 2.3: Partial derivatives of a real valued function; linear approximation and tangent planes; differentiability and the (total) derivative. Students should understand the connection between tangent planes and linear approximation and that differentiability is more than just existence of partial derivatives. The derivative should be defined in general terms, but it will be primarily used for real-valued functions and curves in space. The gradient will be covered in more detail in Section 2.6, so it should only be introduced here as a special case of the derivative.

Lec. 10. Sec. 2.4: Paths and curves. (If more time is required, it can be taken from Section 3.1.)

Lec. 11–12. Sec. 2.5: Properties of the derivative. The Chain Rule should be presented (in its

general matrix form) carefully; not all students will be confident with matrix multiplication.

Lec. 13–14. Sec. 2.6: Gradients and directional derivatives. Students should understand the geometric significance of the gradient and not just the formal computational definition. Note that in this textbook, a distinction is made between the directional derivative *along* a vector and *in the direction of* a vector. The latter requires a unit vector, while the former does not.

Lec. 15. Sec. 3.1: Iterated partial derivatives; showing that a function is a solution to a partial differential equation. (This section may not require a full lecture.)

Lec. 16–17. Sec. 3.3: Extreme values. Attention should be confined to real-valued functions of two variables. Skip the discussion of the Hessian, positive-definiteness, and the general version of the Second Derivative Test in Theorem 5. Instead focus on functions of two variables and the version of the Second Derivative Test in Theorem 6.

Lec. 18–19. Sec. 3.4: The Method of Lagrange Multipliers. The subsection "Several Constraints" is optional. Skip the discussion of the second derivative test for constrained extrema.

Lec. 20. Sec. 4.1: Acceleration and Newton's Second Law. Skip Kepler's Law and the discussion on real-life trajectories.

Lec. 21. Sec. 4.2: Arc length and speed. Briefly introduce arc length as the integral of speed.

Sections 4.3 and 4.4 are covered in Math 20E.

Lec. 22. Sec. 5.1: The double integral as a volume. Iterated integrals as a consequence of Cavalieri's Principle. This section is intended to be an intuitive introduction rather than formal. The actual definitions appear in the next section.

Lec. 23. Sec. 5.2: The double integral over a rectangular region. Stress that the equality of double integrals with iterated integrals is a *theorem* (Fubini's Theorem) and not the definition

Lec. 24. Sec. 5.3: Double integrals over more general regions.

Lec. 25. Sec. 5.4: Changing the order of integration.

Lec. 26. Sec. 5.5: The triple integral. Emphasize that triple integrals are just the natural extension of double integrals. (May be omitted.)