Mathematics 31BH: "Honors Multivariable Calculus" Syllabus (revised September 2016)

Lecture schedule based on:
Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach, fifth edition
by John H. Hubbard and Barbara Burke Hubbard.

Lecture	Section(s)	Topic(s)
1	0.5	The real number system (via infinite decimals). Least upper bound, completeness.
2	0.5	Limits of sequences. Convergence of bounded nondecreasing sequences.
3	1.5	Topological terminology: open and closed sets, closure, interior, boundary. Convergence of sequences in $\mathbb{R}^n$. Uniqueness of limits.
4	1.5	Limits of functions, continuity. Mention the Intermediate Value Theorem 0.5.9, proof optional.
5	1.5	Proposition 1.5.28 on sequential continuity.
6	1.6	Bolzano-Weierstrass Theorem, Extreme Value Theorem.
7	1.6	Mean Value Theorem. Omit the Fundamental Theorem of Algebra.
8	1.7	Definition of derivative as a linear transformation. Partial derivatives, Jacobian matrix.
9	1.7	Differentiability implies that the Jacobian matrix represents the derivative.
10	1.7	Directional derivatives. Gradient vector.
11	1.8	Rules for computing derivatives, emphasizing the Chain Rule.
12	1.9, 2.10	Brief overview: continuity of partials is needed for differentiability; the Inverse Function Theorem. Omit proofs.
13	3.1	Manifolds in $\mathbb{R}^n$, definition and examples.
14	3.1	Parametrizations of manifolds. Definition 3.1.18 can be "relaxed", as seen later in Definition 5.2.3.
15	3.2	Definition of tangent space of a manifold. Computing the tangent space from equations.
16	3.2	Computing the tangent space from a parametrization.
17	3.3	Multivariable Taylor polynomials. Multi-index notation. Equality of mixed partial derivatives.
18	3.3	Taylor's Theorem 3.3.16 without remainder.
19	3.5	Quadratic forms. Completing the square in several variables. Signature of a quadratic form.
20	3.5	Positive/negative definite forms. Invariance of signature. (Non)degenerate forms.
21	3.6	Critical points, second derivative test (Theorem 3.6.8).
22	3.6	Saddle points, degenerate critical points, examples.
23	3.7	Constrained critical points, Lagrange multipliers.
24	3.7	Lagrange multiplier examples. Skip "Classifying Constrained Critical Points". The Spectral Theorem is optional.
25	4.1	Defining the Riemann integral in $\mathbb{R}^n$. Upper and lower integrals, Riemann sums.
26	4.1	Properties of volume in $\mathbb{R}^n$.

Notes:

1. This syllabus is designed for a 1-quarter course with 30 academic hours of instruction. It is sectioned into 26 lectures; this leaves 2 lectures available for in-class midterm exams and 2 lectures for review (or holidays). It is based on the following textbook:

    - *Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach, fourth edition* by John H. Hubbard and Barbara Burke Hubbard.

2. The Math 31H Honors Calculus sequence is a rigorous treatment of multivariable calculus, including linear algebra and differential forms, for a self-selected population of students who have scored a 5 on the Advanced Placement Calculus BC exam. Math 31AH, 31BH, and 31CH substitute respectively for the standard calculus courses 20F (soon to be 18), 20C, and 20E; students who complete the sequence are also exempt from Math 109 due to the emphasis on proof. A minimum grade of B- in each course is required to continue in the sequence. The textbook includes more material than can be covered in three quarters, so it is necessary to be selective, especially about which of the major theorems can be fully proved in class. Scheduling midterm exams outside of class is an option for securing more time for course material. The Honors sequence is more rigorous and theoretically-oriented than the standard calculus sequence, but students should still learn to compute as well as to prove.

    Math 31BH covers the differential calculus of multivariable functions, in the general setting of $\mathbb{R}^n$. Main topics are the derivative as a linear transformation, manifolds in $\mathbb{R}^n$, and local extrema of multivariable functions.

3. Section 0.5 of the textbook covers the structure of the real number system including completeness and limits. This material can be covered in either 31AH or 31BH. It has more continuity (so to speak) with the 31BH calculus content, but in 31AH it can help to build students' familiarity with definitions, reasoning, and proof techniques. In these syllabi it has been included in 31BH.

4. The treatment of multivariable calculus in this course is in the general setting of $\mathbb{R}^n$, in contrast to Math 20C which is restricted to $n = 2$ or 3. Although 31BH students will have a deeper understanding of the concepts, they may experience a language or notational barrier when taking subsequent applied courses such as engineering or physics. Instructors should address the issue of translation across this barrier, for example connecting the derivative defined as a linear transformation with partial derivatives or gradient vectors.

5. Note that Definition 1.5.20 is the "French" definition of limit of a function, specifying $|x-x_0| < \delta$ rather than $0 < |x - x_0| < \delta$. This has the consequence that if $\lim_{x\to a} f(x)$ exists and $f(a)$ is defined, they must be equal. It is recommended that the instructor adopt the standard "U.S." definition instead.

6. Note that the "spherical coordinates" used in Example 3.1.20 and elsewhere are not the standard ones, but are analogous to latitude and longitude.

7. This syllabus optimistically assumes that the last two lectures can introduce the definition of the integral. If not, it can be done in 31CH.