MIDTERM #1: TONIGHT.
7pm in WLH 2001

• No electronic devices allowed.
• Check your assigned seat on TritonEd.
• Bring student ID.
• Do all work on exam paper; scratch paper will be available.
• Have FUN!
(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Consider the following system of linear equations.

\[ \begin{align*}
  x_1 &+ 2x_2 &= 1 \\
  2x_1 &+ x_2 &= -1 \\
  x_1 &- x_2 &= h
\end{align*} \]

(a) Find all value(s) of \( h \) for which the system is consistent, and describe the corresponding solution set.

(b) Find all value(s) of \( h \) for which the system is inconsistent.

(c) Is the corresponding homogeneous system consistent? If so, describe its solution set.
2. (6 points) Let \( A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 2 \\ 3 & 1 & 7 \end{bmatrix} \).

(a) Find the reduced row echelon form of \( A \).

(b) Describe the solution set of the homogeneous equation \( Ax = 0 \).

(c) Let \( b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \). Is the equation \( Ax = b \) consistent? If it is, describe the solution set.
(6 points) 3. For each $k \in \mathbb{R}$, let $S_k$ be the set of vectors in $\mathbb{R}^3$ given by $S_k = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 7 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 9 \\ k \end{bmatrix} \right\}$.

For each of parts (a) - (c), find the value(s) of $k$ for which $S_k$ has the indicated property. Be sure to show how you arrived at each answer.

(a) $S_k$ is linearly independent.

(b) $S_k$ is linearly dependent.

(c) $S_k$ spans $\mathbb{R}^3$. 
4. (6 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$T \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + T \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + T \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$ 

Is $T$ one-to-one? Justify your answer.