Today §6.1/6.7: Inner Products
Next: §6.2: Orthogonality

Reminders:
MyMathLab Homework #7: Due **THURSDAY** by 11:59pm
MATLAB Homework #5: Due **FRIDAY** by 11:59pm

Final Exam is on **Saturday, March 17 11:30am - 2:30pm**
@ the **BEGINNING** of Exam Week.
An $n \times n$ matrix $A$ is diagonalizable if $A = PDP^{-1}$ invertible – diagonal. This is equivalent to $\mathbb{R}^n$ having a basis of eigenvectors for $A$. This happens, e.g., when $A$ has $n$ distinct real eigenvalues. But it can fail.

There are two values: roots of $PA(\lambda) = \text{det}(A - \lambda I)$ – degree $n$ polynomial must have all real roots for $A$ to be diagonalizable.

The algebraic multiplicity of $\lambda$ is the degree of this root in $PA$. The geometric multiplicity of $\lambda$ is $\dim \text{Nul}(A - \lambda I)$.

Always have geometric multiplicity $(\lambda) \leq \text{algebraic multiplicity} (\lambda)$ can be $\neq$.

$A = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 4 \\ 0 & 0 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 0 & -2 \\ 2 & 5 & 5 \\ 0 & 0 & 5 \end{bmatrix}$ $PA(\lambda) = PB(\lambda) = (5 - \lambda)^2 (4 - \lambda)$.

$\dim \text{Nul}(A - SI) = 2$, $\dim \text{Nul}(B - SI) = 1$. 
**Theorem:** If $A$ has a repeated eigenvalue $\lambda$, then 
$$\dim \text{Nul} (A-\lambda I) \leq \text{multiplicity of } \lambda.$$ 
The matrix is diagonalizable if and only if $P_A$ has only real roots, and 
$$\dim \text{Nul} (A-\lambda I) = \text{multiplicity of } \lambda \text{ for each } \lambda.$$ 
In this case, any bases for the eigenspaces together form a basis for $\mathbb{R}^n$.

So, just which matrices are diagonalizable? 
That's a really tough question to answer. A partial answer will be given at the end of the course. To get there...
§ 6.1 **Definition:** The dot product or inner product on $\mathbb{R}^n$ is defined by

$$u \cdot v = \langle u, v \rangle = u^T v$$

**Properties:**
The length of a vector is defined to be

$$\| \mathbf{v} \| = \sqrt{\mathbf{v} \cdot \mathbf{v}}$$

More generally, the distance between two vectors $\mathbf{u}, \mathbf{v}$ is

$$\text{dist}(\mathbf{u}, \mathbf{v}) = \| \mathbf{u} - \mathbf{v} \|$$

E.g., $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$
How does $||u - v||$ relate to $||u||$, $||v||$?
Law of cosines (in 2 dimensions):

\[ a^2 + b^2 = c^2 + 2ab \cos \theta \]
Cauchy - Schwarz Inequality

\[ |u \cdot v| \leq \|u\| \|v\| \]