Today: §1.9: The Matrix of a Linear Transformation
Next: §2.1: Matrix Operations

Reminders:
MyMathLab Homework #3: Due Mon, Jan 29.
MATLAB Homework #2: Due TONIGHT
Midterm 1: Next Wed, Jan 31, 8-10pm.
practice midterms posted on webpage.
seat assignment posted on TritonEd.
A function/ transformation \( T: \mathbb{R}^n \rightarrow \mathbb{R}^m \) is called linear if it respects addition and scalar multiplication.

\[
T(u + v) = T(u) + T(v)
\]

\[
T(cu) = cT(u)
\]

\[
\Rightarrow T(0) = 0
\]

\[
T(c_1v_1 + c_2v_2 + \cdots + c_kv_k) = c_1T(v_1) + c_2T(v_2) + \cdots + c_kT(v_k)
\]
Definition: The standard basis vectors in $\mathbb{R}^n$ are

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, \quad e_n = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.$$ 

E.g. In $\mathbb{R}^2$, there are 2 standard basis vectors.

In $\mathbb{R}^3$, there are 3.

They are the columns of the identity matrix:

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}.$$
Theorem. A linear transformation $T$ is completely determined by its image on the standard basis vectors $T(e_1), T(e_2), \ldots, T(e_n)$. $T: \mathbb{R}^n \to \mathbb{R}^m$

Moreover, every linear transformation is a matrix transformation!
Eg. The “identity” function $T(x) = x$ is linear.
What is its matrix?

Eg. The rotation ccw 45° is linear. (Why?)
What is its matrix?
More function language.

Definition: A transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ is called \textit{onto}.

\textbf{E.g.} Every rotation is both one-to-one and onto.

\textbf{E.g.} $T(x) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} x$
E.g. \( T(x) = \begin{bmatrix} 3 & 1 \\ 5 & 7 \\ 1 & 3 \end{bmatrix} x \)

**Theorem:** Let \( T: \mathbb{R}^n \to \mathbb{R}^m \) be a linear transformation with standard matrix \( A \).

(a) \( T \) maps \( \mathbb{R}^n \) onto \( \mathbb{R}^m \) iff:

(b) \( T \) is one-to-one iff: