1. (a) (5 points) What do we call a benefit premium?

(b) (5 points) What will happen if a company charges only a benefit premium?

(c) (5 points) Explain in this context the sense of security loading: why does the company add it?

2. (5 points) What does the notation $P\left(\overline{A}_{x:n}^{1}\right)$ mean?
3. (10 points) Consider two groups of clients of the same age $x$. In each group the distribution of $T(x)$, the time-until-death, is the same for all clients. However, if $T^{(1)}(x)$ and $T^{(2)}(x)$ are the r.v.’s mentioned for clients from the first and the second group, respectively, then $P(T^{(1)}(x) \geq t) > P(T^{(2)}(x) \geq t)$ for all $t$. Which group is healthier?

For each characteristic below figure out for which group it will be larger. JUSTIFY briefly your answers. (Hint: Since the characteristics below are connected with each other, we do not have to analyze all of them separately.)

(a) $A_x$

(b) $a_x$

(c) $A_{x:m}$

(d) $a_{x:m}$

(e) $P_x$

(f) $P_{x:m}$

(g) $P^1_{x:m}$

(h) $\overline{P}^1_{x:m}$
4. (10 points) Write explicit formulas for \( \bar{P}_x \) and \( P_x \) in the exponential case. (Hint: Recall that in this case the curtate life-time \( K \) has a geometric distribution. More precisely, \( P(K = k) = pq^k \) where \( k = 0, 1, \ldots \), and the parameters \( p = P(K = 0) = 1 - \exp\{-\mu\}, \ q = 1 - p = \exp\{-\mu\} \). (Justify this when writing your solution.))
5. Consider the premiums based on the Equivalence Principle (benefit premiums) in the fully discrete time case for (i) the whole life insurance; (ii) the $n$-year endowment; (iii) the $n$-year pure endowment; (iv) the $n$-year term insurance; (v) the $n$-year deferred whole life annuities.

(a) (5 points) Comparing the cases (i) and (ii), say in which case the premium is larger. Justify your answer.

(b) (5 points) Do the same for the cases (ii), (iii), and (iv).
(c) (15 points) Find the limits of the premiums for all cases for $\delta \to 0$, and $\delta \to \infty$. 
6. (5 points) Show that

\[ nP_x = P^1_{x:n} + P^1_{x:n-1} A_{x+n}, \]

where \( P^1_{x:n} \) is the benefit premium for the pure endowment insurance.
7. A company deals with a portfolio of 1000 polices on 30-year-old people providing a 20-year endowment insurance with a benefit of $50,000. The lifetimes of all clients are independent with a distribution consistent with the Illustrative table. The company proceeds from an interest rate of 4%.

(a) (10 points) Find the benefit premium.

(b) (12 points) Estimate the security loading (in percents) that should be added to the benefit premium for the probability of making a profit to be larger than 0.95. Find the premium itself.
8. (8 points) Let $A_{30:35}^1 = 0.054$, $A_{30:35} = 0.386$, and $\delta = 0.03$. Find the benefit premium $P(\overline{A}_{30:35})$. \textit{(Hint:} First, make notice that $P$ is written without a bar, while $\overline{A}$ in $P(\overline{A}_{30:35})$ has a bar. Second, you can apply the approximation $\overline{A}_x = \frac{e^{\delta x} - 1}{\delta} \overline{A}_x$. However, when dealing with $A_{x:n}^1$ and $A_{x:n}$, we should realize to which quantity we can apply a similar approximation, and to which we cannot. To this end, look up again the formula $A_x = A_{x:n}^1 + nE_xA_{x+n}$, and realize that the quantity $nE_x$ is the same for the discrete and continuous cases as well.)