A List of Index Reduction Formulars

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Let X be a projective variety over the field F. Let G be a connected semisimple adjoint linear algebraic group which acts on X over F such that for the points over the separable closure Fs of $F, G(Fs)$ acts transitively on $X(Fs)$. In [P], the second author computed the Quillen K-theory $K_*(X; D)$ for any such X and any central simple algebra D over F. This leads by the method of Schofield and van den Bergh in [SB, Lemma 1.4] to an index reduction formula, i.e., a formula for the Schur index

$$\mathrm{ind}(D \otimes_F F(X)),$$

where $F(X)$ is the function field of the variety X. The value of $\mathrm{ind}(D \otimes_F F(X))$ is expressed in terms of certain algebras canonically associated to G and the dimensions of certain representations of a parabolic subgroup of the simply-connected cover of G. In [MPW], we have determined the algebras and computed the integers appearing in the general formula from the varieties associated to all the simple algebraic groups G and for products and transfers of such varieties. We give here a list summarizing the resulting explicit index reduction formulas in all cases where G is one of the classical groups, except for the "trialitarian" groups of outer type D_4.

The index reduction formula given here allow one in particular to complete easily the kernel of the Brauer group map $\mathrm{Br}(F) \to \mathrm{Br}(F(X))$. These kernels were determined in a different way, without using K-theory, in [MT].

The function fields $F(X)$ considered here are treated in [KR] from a different perspective; they are generic for the property of G having a parabolic subgroup in a certain conjugacy class defined over the field.

We use the following terminology and notation: If A is a central simple F-algebra, then the *degree* of A is $\deg(A) = \sqrt{\dim_F(A)}$. By Wedderburn's Theorem, $A \cong Ms(B)$ ($= s \times s$ matrices over B) where B is a division algebra. Then, the *Schur index* of A is $\mathrm{ind}(A) = \deg(B) = \deg(A)/s$. Let $A^{\otimes j} = A \otimes_F A \otimes_F \ldots \otimes_F A$($j$ times). Set $\exp(A)$ to be the order of the class of A in the Brauer group $\mathrm{Br}(F)$. Let A^* be the multiplicative group of units of A. We abuse notation by describing varieties in terms of their F-points, where it is to be understood that for fields $L \supseteq F$, the L-points are obtained in the corresponding way after scalar extension. For example, if A

is a central simple F-algebra of degree n, then the Brauer-Severi of A is $B8 = \{I \mid I$ is a right ideal of F with $\dim_F(I) = n\}$ (viewed as a closed subvariety of the projective G rassmannian variety of n-dimensional F-vector subspaces of the n^2-dimensional vector space A). By this we mean, more precisely, that $B8$ is the variety with L-points $B8(L) = \{J \mid J$ is a right ideal of $A \otimes_F L$ with $\dim_L(J) = n\}$.

The 2-adic valuation $v_2 : \mathbb{Z} \to \mathbb{Z}$ will appear in several formulas below. For $n \in \mathbb{Z} n \neq 0$ set $v_2(n) = a$ if $n = 2^a b$ with b odd.

For the terminology associated to quadratic forms and algebras the involution, Scharlau's book [Sch] is a good reference.

We list the index reduction formulas according to the type of the Dynkin diagram of the associated algebraic group (cf. [T1]), Throughout D denotes any central simple algebra over F.

I. Type A_{n-1}, inner type

Let A be any central F-algebra of degree $n, n \geq 2$, and let $G = PGL(A) = A^*/F^*$. For integers $n_i, 1 \leq n_1 < n_2 < \cdots < n_k \leq n-1$, set

$$X(A; n_1, n_2, \ldots, n_k) = \Big\{ (I_1, I_2, \ldots, I_k) \mid I_1 \subseteq I_2 \subseteq \cdots \subseteq I_k, \\ \text{each } I_j \text{ is a right ideal of } A, \text{ and } \dim_F(I_j) = n \cdot n_j \Big\}.$$

Set $d = \gcd(n_1, \ldots, n_k, n)$. Then,

$$\operatorname{ind}\Big(D \otimes_F F(X(A; n_1, \ldots, n_k))\Big) = \gcd_{1 \leq i \leq n} \left(\frac{d}{\gcd(i,d)} \operatorname{ind}\left(D \otimes_F A^{\otimes i}\right)\right).$$

Special cases: (a) If $k = 1, n_1 = 1$, then $X(A; 1)$ is the Brauer-Severi variety of A, and the index reduction formula coincides with the one given in [SB1] by Schofield and van den Bergh (here $d = 1$, so each $d/\gcd(i,d) = 1$).

(b)If $k = 1, n_1 = r, 1 \leq r \leq n$, then $X(A; n)$ is the generic partial splitting variety studied by Blanchet [B] and Schofield and van den Bergh [SB2]. The index reduction formula given here for $F(X(A; n))$ is equivalent to the more complicated one given in [B], and is the same as the one given in [S] and [W].

II. Type A_{n-1}, outer type

Let L be a Galois extension field of F with $[L : F] = 2$, and let τ be the nonidentity F-automorphism of L. Let (A, σ) be a central simple F-algebra A of degree n over $F, n \geq 2$, with an L/F involution σ of the second kind on A. That is, σ is an involution of A with $\sigma|_L = \tau$. Here, $G = \{a \in A^* \mid \sigma(a)a \in F^*\}/L^*$, an algebraic group over F.

When n is even $n = 2m$ there is a *discriminant algebra* $\mathcal{D}(A, \sigma)$ of (A, σ). This $\mathcal{D}(A, \sigma)$ is a central simple F-algebra of degree $\binom{2m}{m}$, exponent 2, and index 1,2, or 4, and is obtainable as follows (cf. [M2,] or

[KMRT,]): Let $\lambda^m(A)$ be the "m-th exterior power algebra" of A, i.e., when A is obtained by Galois descent from $\mathrm{End}_L(V)(\dim_L(V)=m)$, then $\lambda^m(A)$ is obtained by the corresponding descent from $\mathrm{End}_L(\Lambda^m V)$. The involution σ on A induces an L/F involution $\tilde{\sigma}$ of the second kind on $\lambda^m(A)$. But $\lambda^m(A)$ has another involution, p, of the first kind, obtained by Galois descent from the involution on $\mathrm{End}_F(V)$ induced by the extension product pairing $\Lambda^m V \times \Lambda^m V \to \Lambda^{2m} V = F$; this pairing is symmetric if m is even and skew-symmetric if m is odd.

Then $p \circ \hat{\sigma} = \tilde{\sigma} \circ p$ is a τ-semilinear automorphism of $\lambda^m(A)$, and the discriminant algebra is defined as the set of fixed points: $\mathcal{D}(A,\sigma) = (\lambda^m(A))^{p\tilde{\sigma}}$.

Now, for n even or odd, take integers $n_1, \ldots, n_k$ with $1 \le n_1 < n_2 < \cdots < n_k \le n-1$, and $n_1 + n_k = n_2 + n_{k-1} = \ldots = n$. Let

$$X(A,\sigma;n_1,\ldots,n_k) = \Big\{(I_1, I_2, \ldots, I_k) \,|\, I_1 \le I_2 \le \cdots \le I_k \le A, \text{ each } I_i \\ \text{is a right ideal of } A, \dim_L(I_i) = n \cdot n_i, \text{ and } \sigma(I_{k+1-i})I_i = 0\Big\}.$$

Let $d = \gcd(n_1, \ldots, n_k, n)$. Then,

(i) for n odd,

$$\begin{aligned} &\mathrm{ind}\,(D \otimes_F F\,(X(A,\sigma;n_1,\ldots,n_k))) \\ &= \gcd\left(\mathrm{ind}(D), \gcd_{1 \le j \le n-1}\left(2^{\frac{d}{\gcd(j,d)}}\mathrm{ind}\,(D \otimes_F A^{\otimes j})\right)\right). \end{aligned}$$

(ii) For n even, if k is even, set $l = n_{(k/2)+1} - n_{k/2}$, and define

$$n_{\mathcal{D}} = \begin{cases} 1 & \text{if } k \text{ is odd;} \\ 2 & \text{if } k \text{ is even, and } l \text{ is a 2-power or some } v_2(n_i) < v_2(n)\,; \\ 4 & \text{if } k \text{ is even, each } v_2(n_i) \ge v_2(n), \text{ and } l \text{ is not a 2-power.} \end{cases}$$

Then,

$$\begin{aligned} \mathrm{ind}(D \otimes_F F(X(A,\sigma;n_1,\ldots,n_k))) = \gcd\Big(&\mathrm{ind}(D), n_{\mathcal{D}}\mathrm{ind}(D \otimes_F \sigma(A,\sigma)), \\ &\gcd_{1 \le j \le n-1}\left(2^{\frac{d}{\gcd(j,d)}}\mathrm{ind}(D \otimes_F A^{\otimes j})\right)\Big). \end{aligned}$$

III. Type B_n

Assume char $(F) \neq 2$. Let V be a vector space over F with $\dim_F(V) = 2n+1, n \ge 1$, and let $q : V \to F$ be a nondegenerate quadratic form. Here, $G = PSO(q) = SO(q) = \{f = \mathrm{End}_F(V) \,|\, q(f(v)) = q(v), \text{ all } v \in V, \text{ and } \det(f) = 1\}$. Let $C_0(q)$ be the even part of the Clifford algebra of q.

So, $C_0(q)$ is a central simple F-algebra of degree 2^n. A subspace W of V is called totally q-isotropic if $q(w) = 0$ for all $w \in W$.

For integers $n_i, 1 \leq n_1 < n_2 < \cdots < n_k \leq n$, set

$$X(q; n_1, \ldots, n_k) = \{(V_1, \ldots, V_k) \,|\, V_1 \subseteq V_2 \subseteq \cdots \subseteq V_k \subseteq V, \text{ each } V_j \text{ is a totally } q\text{-isotropic subspace of } V, \dim_F(V_j) = n_j\}\,.$$

Then,

$$\operatorname{ind}(D \otimes_F F(X(q; n_1, \ldots, n_k))) = \gcd\left(\operatorname{ind}(D), 2^{n-n_k}\operatorname{ind}(D \otimes_F C_0(q))\right)\,.$$

Special case: Let $k = 1$ and $n_1 = 1$. Then $X(q; 1)$ is the function field of the quadratic hypersurface $q = 0$. The index reduction formula given above for this case is the same as the one given by the first author in [M1, Th. 1] and proved subsequently in a different way by Tignol in [Ti, Cor. 4.1].

IV. Type C_n

Let (A, σ) be a central simple algebra A of degree $2n$ over F ($\operatorname{char}(F) \neq 2$), $n \geq 2$, with a symplectic type involution σ of the first kind on A. Here, $G = \{a \in A^* \,|\, \sigma(a)a \in F^*\}/F^*$.

For integers $n_i, 1 \leq n < n_2 < \cdots < n_k \leq n$, let

$$X(A, \sigma; n_1, \ldots, n_k) = \{(I_1, \ldots, I_k) \,|\, I_1 \leq I_2 \leq \cdots \leq I_k \leq A, \text{ each } I_j \text{ is a right ideal of } A \text{ with } \sigma(I_j)I_j = 0 \text{ and } \dim_F(I_j) = 2n \cdot n_j\}\,,$$

Set $d = v_2(n_1, \ldots, n_k, 2n)$. Then,

$$\operatorname{ind}(D \otimes_F F(X(A, \sigma; n_1, \ldots, n_k))) = \gcd\left(\operatorname{ind}(D), 2^d\operatorname{ind}(D \otimes_F A)\right)\,.$$

V. Type D_n

Let (A, σ) be a central simple algebra A of degree $2n$ over F (char $(F) \neq 2$), with an orthogonal type involution σ of the first kind on A. Associated to σ there is a (signed) discriminant $\operatorname{disc}(A, \sigma) \in F^*/F^{*2}$ and an even Clifford Algebra $C_0(A, \sigma)$ (cf. [J], [T2], [KMRT]). When A is split, $A \cong \operatorname{End}_F(V)$ where $\dim_F(V) = 2n$, then σ is the involution σ_q on $\operatorname{End}_F(V)$ associated to some nondegenerate quadratic form $q : V \to F; q$ is uniquely determined up to a scalar multiple. We have $C_0(\operatorname{End}_F(V), \sigma_q) = C_0(q)$, the even part of the Clifford algebra of q, and $\operatorname{disc}(\operatorname{End}_F(V), \sigma_q) = d_I(q)$ the signed determinant of q. When A is not split, $\operatorname{disc}(A, \sigma)$ is obtainable by Galois descent from $\operatorname{disc}(\operatorname{End}_F(V), \sigma_q)$. More directly, as shown in [KPS, §2], take any $a \in A^*$ with $\sigma(a) = -a$; then $\operatorname{disc}(A, \sigma) = (-1)^n\operatorname{Nrd}(a) \in F^*/F^{*2}$, where Nrd is the reduced norm. $C_0(A, \sigma)$ is obtainable by Galois descent from $C_0(\mathrm{End}_F(V), \sigma_q)$, but a "national" construction of $C_0(A,\sigma)$ without using descent is also possible, cf. [T2], [KMRT]. When $\mathrm{disc}(A,\sigma) \neq 1 \in F^*/F^{*2}$ $C_0(A,\sigma)$ is a central simple algebra of degree 2^{n-1} over $L = F(\sqrt{\mathrm{disc}(A,\sigma)})$. When $\mathrm{disc}(A,\sigma) = 1, C_0(A,\sigma) = C_+(A,\sigma) \oplus C_-(A,\sigma)$, where each of C_+, C_- is a central simple F-algebra of degree 2^{n-1}. Also, $C_+(A,\sigma) \otimes_F C_-(A,\sigma)^{op} \sim A$ in $\mathrm{Br}(F)$, cf. [T2, Prop. 7], [T3, 6.2] or [KMRT].

Here, $G = \{a \in A \mid \sigma(a)a = \lambda \in F^* \text{ and } \mathrm{Nrd}\ (a) = \lambda^n\}/F^*$. (If $\sigma(a)a = \lambda \in F^*$, then $\mathrm{Nrd}\ (a) = \pm\lambda^n$. Thus, G is the connected component of the identity of the automorphism group $\mathrm{Aut}(A,\sigma)$.) G is of inner type iff $\mathrm{disc}(A,\sigma) = 1$. For $1 \leq i \leq n$, let

$$q_i = \{I \mid I \text{ is a right ideal of } A \text{ with } \sigma(I)\cdot I = 0 \text{ and } \dim_F(I) = 2n\cdot i\}\,.$$

Then, for $1 \leq i \leq n-1, G(Fs)$ acts transitively on $q_i(Fs)$, but there are two orbits in the action of $G(Fs)$ on $q_n(Fs)$. Let q^+ and q^- be the irreducible components of the variety qn which correspond to the two G-orbits.

(i) Assume first that $\mathrm{disc}(A,\sigma) = 1$, i.e., G is of inner type. For integers $n_i, 1 \leq n_1 < n_2 < \cdots < n_k \leq n-1$, set

$$\begin{aligned} X(A,\sigma; n_1,\ldots,n_k) = \{I_1, I_2, \ldots, I_k) \mid I_1 \subseteq I_2 \subseteq \cdots \subseteq I_k \subseteq A \\ \text{and each } I_1 \in qn_j\}\,. \end{aligned}$$

Let $d = v_2(\gcd(n_1,\ldots,n_k,2n))$, and let

$$s = \begin{cases} 2^d & \text{if } n \text{ is even or } n_k < n-1\,; \\ 1 & \text{if } n \text{ is odd and } n_k = n-1 (\text{so } 2^d = 2 \text{ or } 1)\,, \end{cases}$$

Then,

$$\begin{aligned} \mathrm{ind}(D \otimes_F F(X(A,\sigma;n_1,\ldots,n_k))) = \gcd\Big(\mathrm{ind}(D), s\ \mathrm{ind}(D\otimes_F A), \\ 2^{n-n_k-1}\mathrm{ind}(D \otimes_F C_+(A,\sigma)), 2^{n-n_k-1}\mathrm{ind}(D\otimes_F C_-(A,\sigma))\Big)\,. \end{aligned}$$

For $1 \leq n_1 < n_2 \cdots < n_{k-1} \leq n-2$, let $n_k = n$, and set

$$\begin{aligned} X(A,\sigma;n_1,\ldots,n_{k-1},n,+) &= \{(I_1,\ldots,I_k) \mid I_1 \subseteq I_2 \subseteq \cdots \subseteq I_k \subseteq A, \\ & \qquad I \in q, \ldots, I_{k-1} \in q_{k-1}, I_k \in q^+\}, \\ X(A,\sigma;n_1,\ldots,n_{k-1},n,-) &= \{(i_1,\ldots,I_k) \mid I_1 \subseteq I_2 \subseteq \cdots \subseteq I_k \subseteq A, \\ & \qquad I_1 \in q_1, \ldots, I_{k-1} \in q_{k-1}, I_k \in q^-\}\,. \end{aligned}$$

Then, with $d = v_2(\gcd(n_1,\ldots,n_{k-1},n))$,

$$\begin{aligned} \mathrm{ind}(D\otimes_F F(X(A,\sigma;n_1,\ldots,n_{k-1},n,+))) = \gcd(\mathrm{ind}(D), 2^d\mathrm{ind}(D\otimes_F A), \\ 2^d\mathrm{ind}(D\otimes_F C_+(A,\sigma)), \mathrm{ind}(D\otimes_F C_-(A,\sigma))) \end{aligned}$$

The formula for $\mathrm{ind}(D\otimes_F F(X(A,\sigma;n_1,\ldots,n_{k-1},n,-)))$ is like that for the $+$ case but with $+$ and $-$ interchanged.

(ii) Now assume $\mathrm{disc}(A,\sigma) \neq 1 \in F^*/F^{*2}$, i.e., G is of outer type. Let $L = F(\sqrt{\mathrm{disc}(A,\sigma)}) =$ center of $C_0(A,\sigma)$. For integers $n_i, 1 \leq n_1 < n_2 < \cdots < n_k \leq n-1$, let $X(A,\sigma; n_1, \ldots, n_k)$ be defined as in the inner type case. Let $d = v_2(\gcd(n_1, \ldots, n_k, 2n))$. Then, if n is even or $n_k \leq n-2$, or some n_i is odd,

$$\mathrm{ind}(D \otimes_F F(X(A,\sigma; n_1, \ldots, n_k))) = \gcd(\mathrm{ind}(D), 2^d(\mathrm{ind}(D \otimes_F A), 2^{n-n_k}\mathrm{ind}(D \otimes_F C_0(A,\sigma))) .$$

In the exceptional case where n is odd, $n_k = n-1$, and all n_i are even,

$$\mathrm{ind}(D \otimes_F F(X(A,\sigma; n_1, \ldots, n_k))) = \gcd(\mathrm{ind}(D), 2\mathrm{ind}(D \otimes_F A \otimes_F 2), 2\mathrm{ind}(D \otimes_F C_0(A,\sigma))) .$$

Special cases: (a) Let $k = 1$ and $n_1 = 1$. Then, $X(A,\sigma;1) = \{I | I$ is a right ideal of $A \dim_F(I) = 2n$, and $\sigma(I) \cdot I = 0\}$, which is the involution variety of (A,σ) defined by D. Tao in [Ta]. The index reduction formula for $F(X(A,\sigma;1))$ was computed by Tao in the inner and also the outer type case in [Ta,].

(b) Let $q : V \to F$ be a nondegenerate quadratic form on a vector space V over $F, \dim_F(V) = 2n$. Then the preceding formulas apply to the algebra with involution $(\mathrm{End}_F(V), \sigma_q)$. But they can be restated directly in terms of q as follows. Here $G = \{f \in \mathrm{End}_F(V)^* |$ there is $\lambda \in F^*$ with $q(f(v)) = \lambda q(v)$ all $v \in V$, and $\det(f) = \lambda^n\}/F^*$. G is inner iff $d_I(q) = 1 \in F^*/F^{*2}$. For $1 \leq i \leq n$, let $\mathcal{F}_i = \{W | W$ is a totally q-isotropic F-subspace of V and $\dim_F(W) = i\}$.

Then $\mathcal{F}_i \cong f_i$ as varieties over F, via the Morita correspondence between F-subspaces of V and right ideals of $\mathrm{End}_F(V)$. Let $\mathcal{F}^+$ and $\mathcal{F}^-$ be the irreducible components of $\mathcal{F}_n$, which correspond to the two orbits in the action of $G(Fs)$ on $\mathcal{F}_n(Fs)$. The varieties $X(q; n, \ldots, n_k)$ and $X(q; n_1, \ldots, n_{n-1}, n, \pm)$ one defined by replacing in $X(A,\sigma;\ldots)$ right ideal of A in f_i by F-subspaces W of V in $\mathcal{F}_i$. In the index reduction formulas the terms involving A are redundant and can be deleted, and $C_0(A,\sigma)$ (resp. $C_+(A,\sigma), C_-(A,\sigma)$) is replaced by $C_0(q)$ (resp. $C(q), C(q)$ since when $d_\pm(q) = 1$, each simple component of $C_0(q)$ is similar to the full Clifford algebra $C(q)$ in $\mathrm{Br}(F)$). So, for example, for G of inner type and $1 \leq n_1 \leq n_2 < \cdots < n_k \leq n-1$, then

$$X(q; n_1, \ldots, n_k) = \{(W_1, \ldots, W_k) | W_1 \subseteq W_2 \subseteq \cdots \subseteq W_k \subseteq V \text{ and each } W_j \in \mathcal{F}_j\} ,$$

and

$$\mathrm{ind}(D \otimes_F F(X(q; n_1, \ldots, n_k))) = \gcd(\mathrm{ind}(D), 2^{n-n_k-1}\mathrm{ind}(D \otimes_F C(q))) .$$

Likewise,

$$\begin{aligned}\operatorname{ind}(D\otimes_F F(X(q;n_1,\ldots,n_{k-1},n,+))) &= \operatorname{ind}(D\otimes_F F(X(q;\\ &\qquad n_1,\ldots,n_{k-1},n,-)))\\ &= \gcd(\operatorname{ind}(D),\operatorname{ind}(D\otimes_F C(q)))\,.\end{aligned}$$

Note that in this situation with A split the special case which arose above (n odd, $n_k = n-1$, all n_i even, for G of inner or outer type) no longer yields a different index reduction formula from the other cases.

(c) In the particular case of (b) with $k=1, n=1$, the variety $X(q;1)$ is the projective quadratic hypersurface $q=0$, and the index reduction formulas for inner and outer type cases agree with those given by the first author in [M, Th. 2, Th. 3] (see also [Ti, Cor. 4]).

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