1. Compute the following double integrals:

(a) $\int_1^2 \int_0^1 (1+4xy)dxdy$

(b) Integrate the function $f(x,y) = 2 - x - 2y$ over the region D bounded by the line $y = x - 1$ and by the parabola $y^2 = 2x + 6$. (hint: compute the intersections between the line and the parabola, and draw a picture of the region D).

2. Compute the area of the following surfaces:

(a) The part of the plane $2x + 5y + z = 10$ which lies in the first octant $x \geq 0$, $y \geq 0$ and $z \geq 0$.

(b) The part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy-plane.

(c) The part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(d) The part of the sphere $x^2 + y^2 + z^2 = 4$ which lies above the cone $z^2 = x^2 + y^2$.

3. Use the given transformation to evaluate the following integral:

(a) $\int\int_R \ln(x^2 + y^2)\ dA$, where R is the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ using polar coordinates $x = \rho\cos\theta$ and $y = \rho\sin\theta$.

(b) $\int\int_R x\ dA$, where R is the region bounded by the ellipse $9x^2 + 4y^2 = 36$ with $x = 2u$ and $y = 3v$.

Important formulas:

Surface area of surface given by parametrization $\Phi(u,v)$ with (u,v) in a given domain D:

$$area = \int\int_D |\Phi_u \times \Phi_v|\ dudv.$$

Integral of a *function* f over a parametrized surface S:

$$\int\int_S f(x,y,z)\ dS = \int\int_D f(\Phi(u,v))|\Phi_u \times \Phi_v|\ dudv.$$

Integral of a *vector field* $\mathbf{F}$ over a parametrized surface S:

$$\int\int_S \mathbf{F}(x,y,z)\ d\mathbf{S} = \int\int_D \mathbf{F}(\Phi(u,v)) \cdot (\Phi_u \times \Phi_v)\ dudv.$$

Change of variable formula: See Section 15.6 (page 880) in the Shenk hand-out (downloadable as a pdf file) listed under 'Texts' on the course webpage, or look it up in Stewart, Section 15.9.