

Justify your answers!

1. Is the vector $\mathbf{b} = (1, -3, 1)$ in the span of the vectors $\mathbf{a}_1 = (1, 2, 2)$ and $\mathbf{a}_2 = (3, 1, 1)$?

2. Compute all solutions of $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 3 & 1 \\ 4 & 5 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$.

3. Let $\mathcal{B} = \{x + 1, x - 1\}$. Compute the coordinate vector $[2x + 6]_{\mathcal{B}}$.

more problems on backside

4. (a) Compute the LU -decomposition $A = LU$ of $A = \begin{bmatrix} 2 & 2 \\ 4 & 2 \end{bmatrix}$.

(b) Show for an arbitrary $n \times n$ matrix A that $\det(A) = u_{11}u_{22} \cdots u_{nn}$, the product of the diagonal entries of the matrix U in the LU -decomposition.

5. Compute a basis for the column space of the matrix $A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 7 & 4 \\ 3 & 9 & 6 & 6 \end{bmatrix}$. What is its rank?