MATH 21D MIDTERM SPRING 2002 **Name:** **Section :**

Justify your answers! Put all the essential steps of your solution on this sheet!

1. Do the following series converve or diverge? Why?
   (a) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$ (b) $\sum_{n=1}^{\infty} \frac{\sin(n^3)}{n^3}$

2. Compute the radius of convergence *and* the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$.

3. Solve the initial value problem $y'' - 3y' + 2y = 0$ with $y(0) = 2$ and $y'(0) = 3$.

4. (a) Solve the initial value problem $y' = (1 - 2x)y^2$ with $y(1) = 1/2$.
   (b) Determine the interval for which the solution in (a) is defined.

5. (a) Express the function $f(x) = \frac{1}{1+x^2}$ as a power series in x.
   (b) Express the function $\ln(1 + x^2)$ as a power series in x.