EXERCISES MATH 202B - Ninth Assignment

1. Let $s_\lambda(x_1,\ ...,\ x_N)$ be the Schur function corresponding to the Young diagram λ. Then also the product $s_\lambda(x_1,\ ...,\ x_N)(x_1 +\ ...\ + x_N)$ is a symmetric function and hence must be a linear combination of Schur functions. Calculate this linear combination. (*Hint*: Multiply by Δ and use results about antisymmetric functions).

2. (a) Let e_λ be a minimal idempotent in $(\mathbf{C}S_n)_\lambda$ and let μ be a Young diagram with $n+1$ boxes. Calculate $\chi_\mu(e_\lambda)$, with e_λ viewed as an element in $\mathbf{C}S_{n+1}$. (*Hint* : Caclulate $Tr_{V^{\otimes n+1}}(e_\lambda d)$ for d a diagonal matrix with eigenvalues $x_1,\ ...,\ x_N$; how is this related to $Tr_{V^{\otimes n}}(e_\lambda d)$?)
(b) How does the simple S_{n+1}-module S^μ decompose as a direct sum of simple S_n-modules? Do NOT use the Murnaghan-Nakayama rule stated below.

   We shall prove the *Murnaghan-Nakayama rule* this coming week. This helps to calculate characters of S_n as follows: Let π be a permutation, and let π' be the permutation obtained from π by removing an h-cycle. Then we have

$$\chi_\lambda(\pi) = \sum_\mu (-1)^{r(\mu)-1}\chi_\mu(\pi');$$

   here the summation goes over all Young diagrams μ which can be obtained from λ by removing a rim hook of length h from λ and $r(\mu)$ is the number of rows of the rim hook.

3. Calculate the S_{11} character $\chi_\lambda((1234)(567))$ for $\lambda = [6, 4, 1]$ (unlisted numbers remain fixed in the definition of the permutation).