EXERCISES MATH 202C - Sixth Assignment

1. (a) Calculate a system of generators for $k[x,y]^G$, where the action of a generator of $G \cong \mathbf{Z}/3$ is given by the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$.

   (b) Calculate its Hilbert series.

2. The polynomial $f(x,y) = x^8 + 2x^6y^2 - x^5y^3 + 2x^4y^4 + x^3y^5 + 2x^2y^6 + y^8$ is in the algebra generated by the polynomials $x^2+y^2, x^3y - xy^3, x^2y^2$ (this is $k[x,y]^{\mathbf{Z}/4}$ with the action as given in class). Express f as a polynomial in the generators.

3. Show that

$$\sum_{\alpha \in \mathbf{N}^n} \lambda^\alpha z^{|\alpha|} = \prod_{i=1}^{n} \frac{1}{1 - z\lambda_i} = \frac{1}{det(I - zA)},$$

   where A is an $n \times n$ matrix with eigenvalues $\lambda_1, \ ..., \ \lambda_n$.

4. One can show that $k[x,y]^G$, with the action of the group $G \cong \mathbf{Z}/4$ given by the matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is generated by the polynomials $a = x^2 + y^2$, $b = x^3y - xy^3$ and $c = x^2y^2$. Calculate a Gröbner basis of the ideal of relations, i.e. of the kernel of the map from $k[a,b,c]$ onto $k[x,y]^G$ given by the formulas above.