

Diffusion Approximation for a Heavily Loaded Multi-User Wireless Communication System with Cooperation

Ruth J. Williams
Joint work with Sumit Bhardwaj

Communication System

Queueing Model

Heavy Traffic

Diffusion Approximation

Communication System

Cellular Wireless

Characteristics

Queueing Schematic

Capacity Region

Normalization

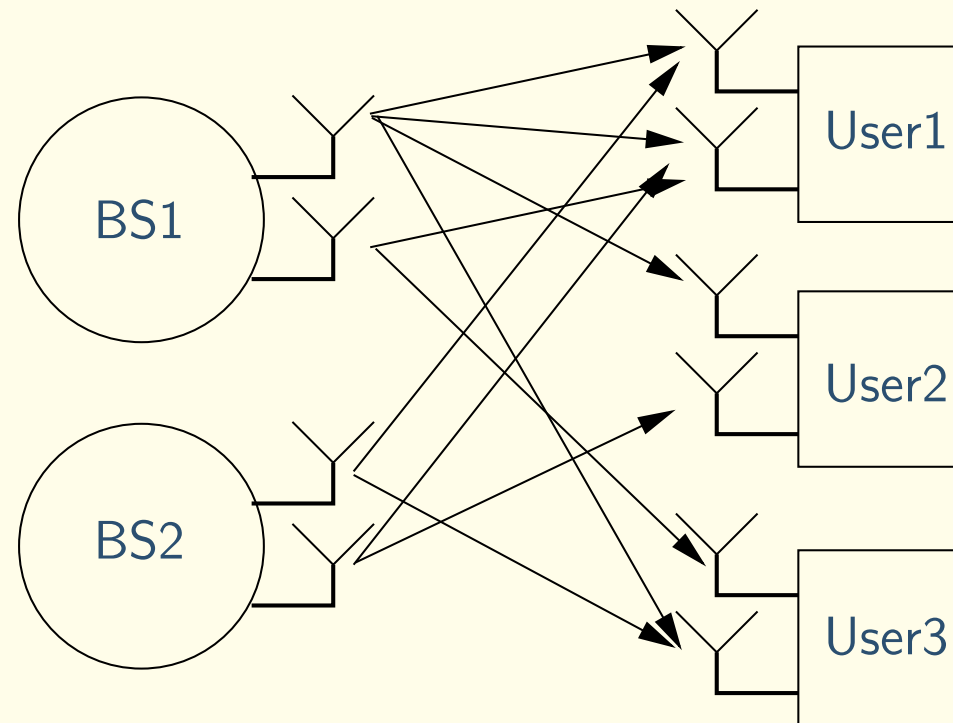
Queueing Model

Heavy Traffic

Diffusion

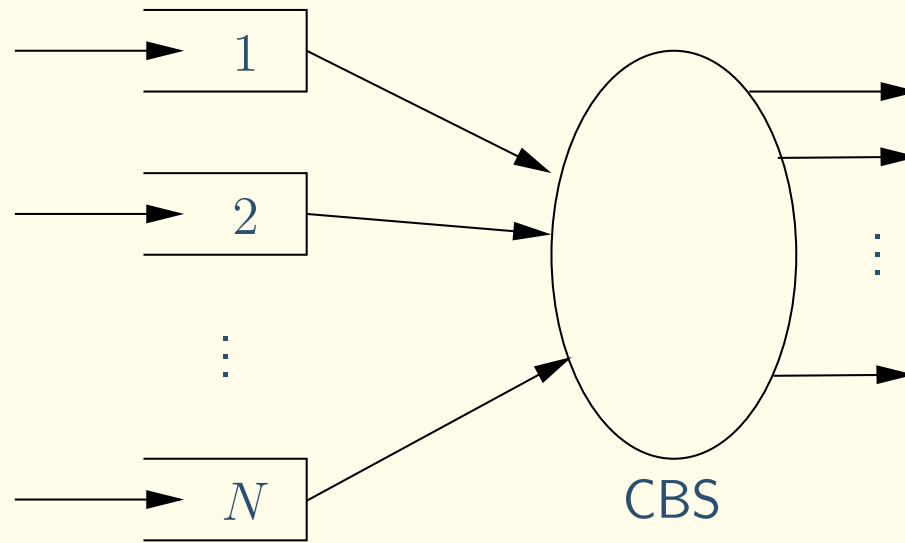
Approximation

Communication System

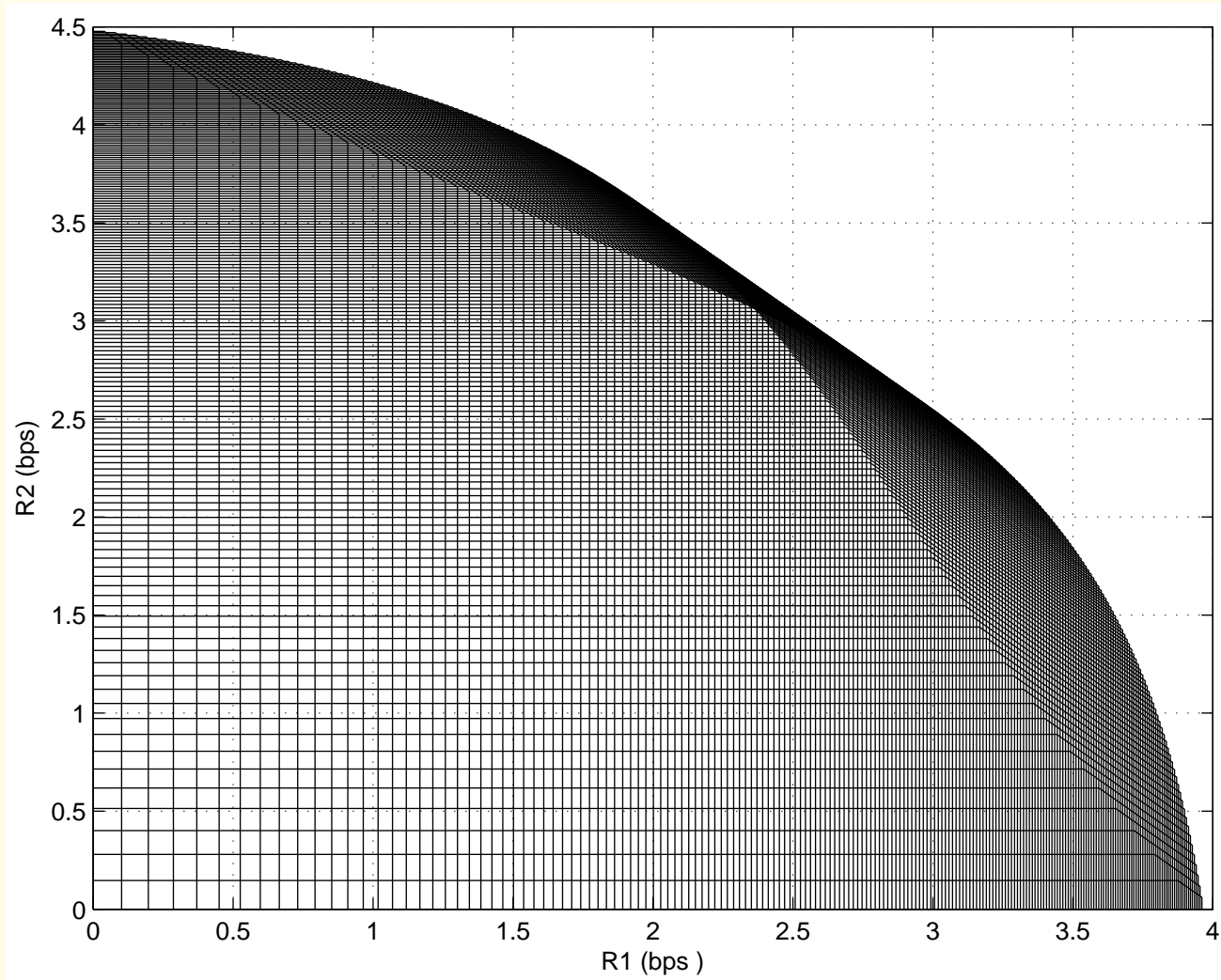


- Downlink (base station to mobiles) modeled as multi-user MIMO broadcast channel
- Multiple cooperating base stations
- **Total channel capacity is enhanced by cooperation** (can be achieved by dirty paper coding)
- Packet-based traffic
- Channel fixed over the period of interest

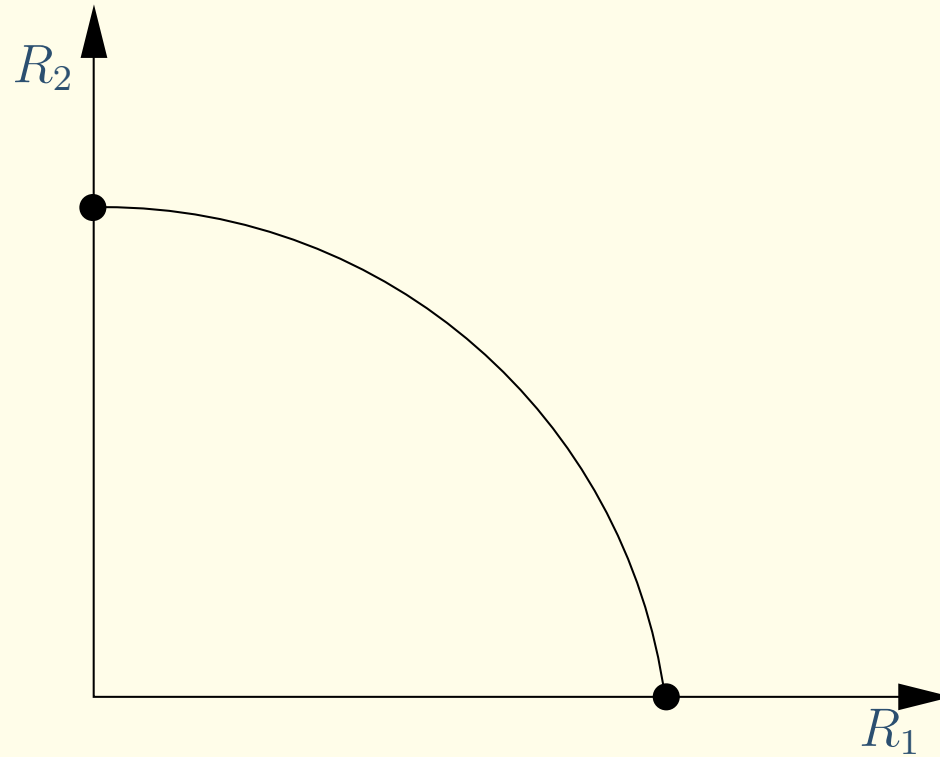
Queueing Schematic



Capacity Region for 2-Users



Assume that cooperation is expressed in terms of sums of rates



Communication
System

Queueing Model

Queueing Schematic

Primitives

Workload Process

Example

Service Policy

Cooperation

Assumptions

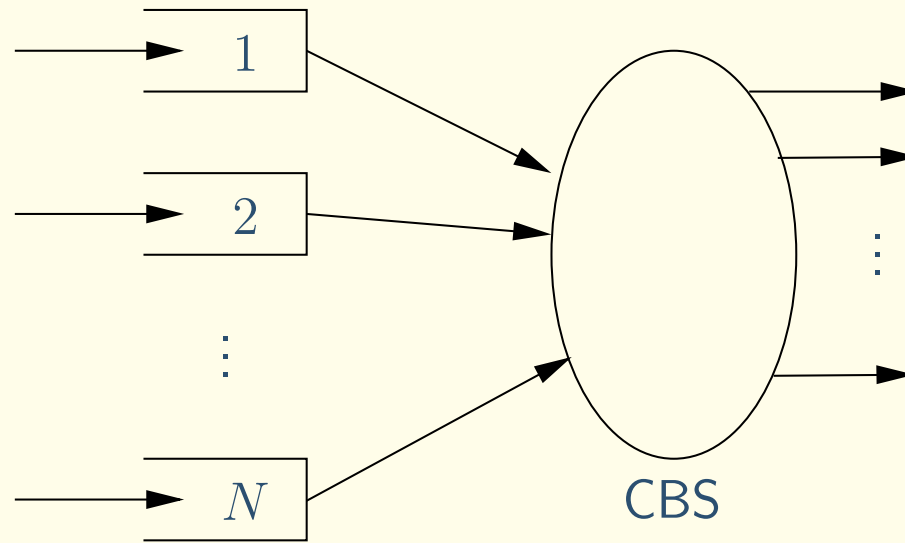
Performance?

Heavy Traffic

Diffusion

Approximation

Queueing Model



- N users
- $E_i(t)$ number of packet arrivals for user i up to t
 - renewal process with rate λ_i
- $V_i(n)$ sum of first n packet lengths (in bits) for user i
 - i.i.d. packet lengths with mean m_i
- $E_i, V_i, i = 1, \dots, N$ are all mutually independent
- Nominal bit arrival rate vector $b = (b_1, \dots, b_N)$ is known
- System starts empty

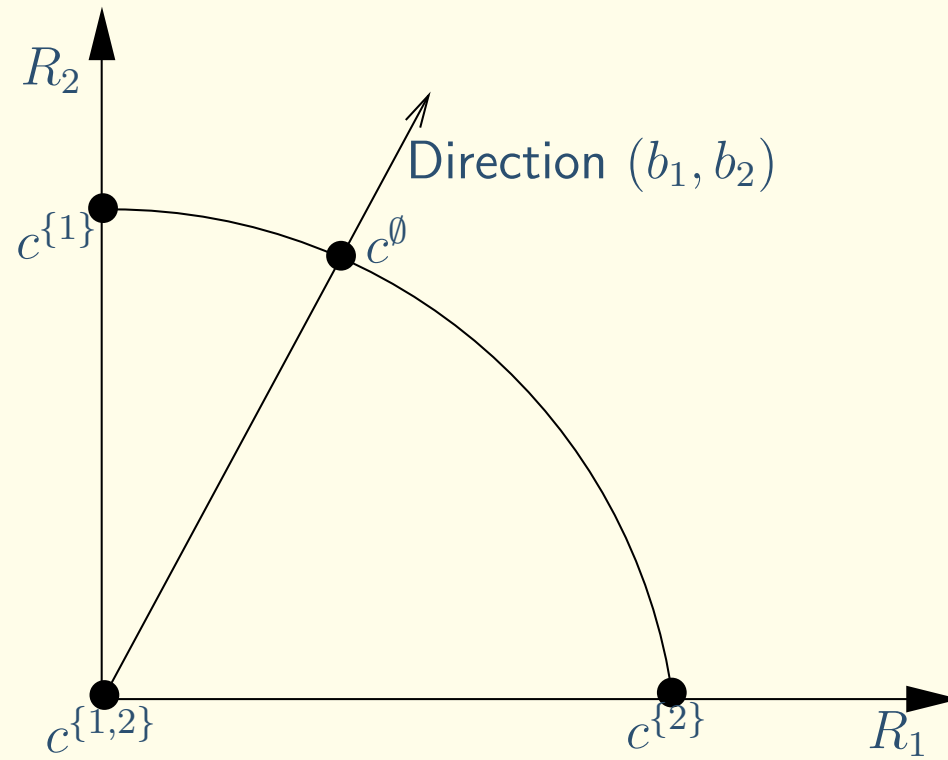
- Workload (number of bits waiting to be transmitted) for the i -th queue at time t :

$$W_i(t) = V_i(E_i(t)) - T_i(t)$$

- Total amount of service (measured in bits) given to the i -th queue up to time t :

$$T_i(t) = \int_0^t \Lambda_i(W(s)) ds$$

where $\Lambda : \mathbb{R}_+^N \rightarrow \mathbb{R}_+^N$ specifies the service rate for each queue as a function of the workload



Define

$$\begin{aligned}\mathcal{K}(w) &= \{i : w_i = 0\} \quad \text{for } w \in \mathbb{R}_+^N \\ \Lambda(w) &= c^{\mathcal{K}(w)}\end{aligned}$$

where for each $\mathcal{K} \subseteq \{1, \dots, N\}$,

$$\begin{aligned}c_i^{\mathcal{K}} &= 0 \quad \text{if } i \in \mathcal{K} \\ c_i^{\mathcal{K}} &= \delta_{\mathcal{K}} b_i \quad \text{if } i \notin \mathcal{K}\end{aligned}$$

where $\delta_{\mathcal{K}} > 0$ is as large as possible while respecting the constraint that $c^{\mathcal{K}}$ is in the capacity region

- The vector c^\emptyset has the (strict) maximum sum of rates (corresponding to maximum cooperation)

$$\sum_i c_i^\emptyset > \sum_i c_i^\mathcal{K} \quad \text{for all } \mathcal{K} \neq \emptyset$$

- The service rate for a fixed non-empty queue is least when all queues are non-empty

$$c_i^\emptyset \leq c_i^\mathcal{K} \quad \text{for all } i \notin \mathcal{K} \text{ and } \mathcal{K} \neq \emptyset$$

- No closed-form expression for the performance of the policy
- Here we consider performance when the system is heavily loaded
- Seek a diffusion approximation for the workload process

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Assumptions

Scaling and FCLT

Diffusion Scaled

Workload

Example

Diffusion

Approximation

Heavy Traffic

- Heavy traffic (simplest case):

$$b_i = \lambda_i m_i = c_i^{\emptyset} \quad \text{for } i = 1, \dots, N$$

- Finite second moments for i.i.d. interarrival times and packet lengths:

$$\alpha_i^2 = \text{SCV for interarrivals times for queue } i$$

$$\beta_i^2 = \text{SCV for packet lengths for queue } i$$

- Diffusion scaling: let $r \rightarrow \infty$ through a sequence and

$$\hat{W}^r(t) = \frac{W(r^2t)}{r}$$

$$\hat{E}^r(t) = \frac{1}{r} (E(r^2t) - \lambda r^2t)$$

$$\hat{V}^r(t) = \frac{1}{r} (V(r^2t) - mr^2t)$$

- Standard FCLT for stochastic primitives:

$$(\hat{E}^r, \hat{V}^r) \Rightarrow (\tilde{E}, \tilde{V}) \quad \text{as } r \rightarrow \infty$$

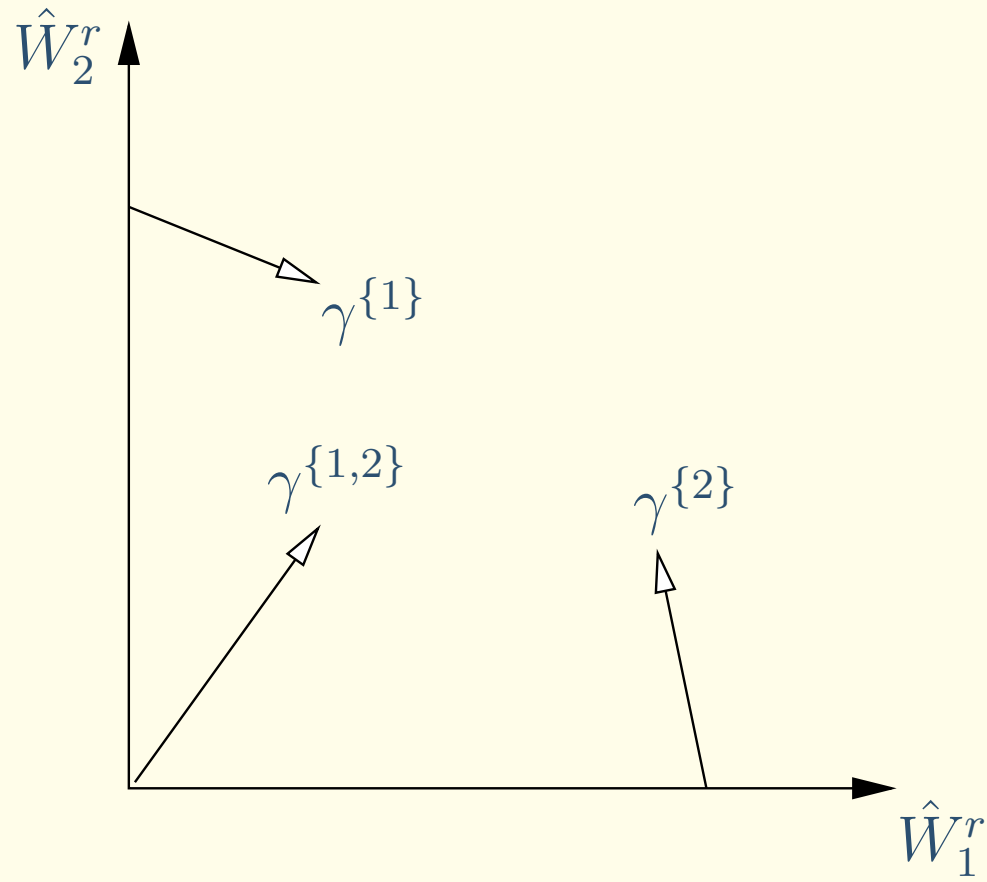
where \tilde{E} and \tilde{V} are independent N -dimensional Brownian motions with non-degenerate diagonal covariance matrices $\Gamma_E = \text{diag}(\lambda_i \alpha_i^2)$ and $\Gamma_V = \text{diag}(m_i^2 \beta_i^2)$

$$\begin{aligned}\hat{W}_i^r(t) &= \frac{1}{r}V_i(E_i(r^2t)) - \frac{1}{r}T_i(r^2t) \\ &= \hat{X}_i^r(t) + \sum_{\mathcal{K} \neq \emptyset} \gamma^{\mathcal{K}} \hat{U}^{r,\mathcal{K}}(t)\end{aligned}$$

where

$$\begin{aligned}\hat{X}_i^r(t) &= \hat{V}_i^r(\bar{E}_i^r(t)) + m_i \hat{E}_i^r(t) \\ \hat{U}^{r,\mathcal{K}}(t) &= \frac{1}{r} \int_0^{r^2t} 1_{\{\mathcal{K}(W(s))=\mathcal{K}\}} ds \\ \gamma^{\mathcal{K}} &= c^\emptyset - c^{\mathcal{K}} \quad \text{for } \mathcal{K} \neq \emptyset\end{aligned}$$

Note, $\gamma_i^{\mathcal{K}} > 0$ if $i \in \mathcal{K}$ and $\gamma_i^{\mathcal{K}} \leq 0$ if $i \notin \mathcal{K}$



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Heavy Traffic

**Diffusion
Approximation**

Tightness

Key Calculation

Pushing on Faces

Limit Theorem

Summary

Diffusion Approximation

Theorem 1. *The sequence of diffusion scaled processes $\{(\hat{W}^r(\cdot), \hat{X}^r(\cdot), \hat{U}^r(\cdot))\}$ is C -tight and any weak limit point $(\tilde{W}, \tilde{X}, \tilde{U})$ defines a semimartingale reflecting Brownian motion in \mathbb{R}_+^N of the form*

$$\tilde{W}(t) = \tilde{X}(t) + \sum_{\mathcal{K} \neq \emptyset} \gamma^{\mathcal{K}} \tilde{U}^{\mathcal{K}}(t), \quad t \geq 0,$$

where \tilde{X} is an N -dimensional driftless Brownian motion with covariance matrix $\Gamma = \text{diag}(\lambda_i m_i^2 (\alpha_i^2 + \beta_i^2))$ and each $\tilde{U}^{\mathcal{K}}$ is a continuous, non-decreasing one-dimensional process that can increase only when \tilde{W} is on the face

$$F_{\mathcal{K}} = \{w \in \mathbb{R}_+^N : w_i = 0 \text{ for all } i \in \mathcal{K}\}.$$

(Proof uses invariance principle of Kang-W '07)

For $\emptyset \neq \mathcal{L} \subseteq \mathcal{K}$,

$$\begin{aligned}n^{\mathcal{K}} \cdot \gamma^{\mathcal{L}} &= \sum_{i \in \mathcal{K}} \gamma_i^{\mathcal{L}} \\&= \sum_{i \in \mathcal{K}} (c_i^{\emptyset} - c_i^{\mathcal{L}}) \\&= \sum_i (c_i^{\emptyset} - c_i^{\mathcal{L}}) - \sum_{i \notin \mathcal{K}} (c_i^{\emptyset} - c_i^{\mathcal{L}}) \\&\geq \sum_i (c_i^{\emptyset} - c_i^{\mathcal{L}}) \\&> 0\end{aligned}$$

because of cooperation

Theorem 2. For each \mathcal{K} such that $|\mathcal{K}| \geq 2$ and each $\emptyset \neq \mathcal{L} \subseteq \mathcal{K}$, we have

$$\int_0^\infty 1_{F_{\mathcal{K}}}(\tilde{W}(s)) d\tilde{U}^{\mathcal{L}}(s) = 0 \text{ almost surely.}$$

Consequently, almost surely,

$$\tilde{W}(t) = \tilde{X}(t) + \sum_{i=1}^N \gamma^{\{i\}} \tilde{U}^{\{i\}}(t), \quad t \geq 0.$$

(Proof uses an extension of an argument given by Reiman-W '88)

Theorem 3. *As $r \rightarrow \infty$, the diffusion scaled process \hat{W}^r converges in distribution to a semimartingale reflecting Brownian motion \tilde{W} in \mathbb{R}_+^N of the form*

$$\tilde{W} = \tilde{X} + \sum_{i=1}^N \gamma^{\{i\}} \tilde{U}^{\{i\}}$$

where the directions of reflection $\{\gamma^{\{i\}}\}_{i=1}^N$ determine a reflection matrix of Harrison-Reiman type $(I - P')$

- The policy has $2^N - 1$ points of operation
- Heavy traffic diffusion approximation
 - ◆ N -dimensional SRBM
 - ◆ Key result: Only reflections on $N - 1$ -dimensional faces matter

Thank You!

We consider a model for a cellular wireless communication system in which data is transmitted to multiple users over a common channel. For information theoretic reasons, the rate of transmission over this channel can be enhanced by cooperation. Assuming a fixed channel and that the average arrival rate of data for each user is known, we consider a simple scheduling policy which exploits cooperation and which has been shown to be throughput-optimal under Markovian assumptions. As a measure of performance under this policy, we establish a heavy traffic diffusion approximation for the workload process. This diffusion process is a semimartingale reflecting Brownian motion (SRBM) living in the positive orthant of N -dimensional space (where N is the number of users). Nominally, this SRBM has one direction of reflection associated with each of the $2^N - 1$ boundary faces. However, we show that in fact only those directions associated with the $(N-1)$ -dimensional boundary faces matter in the heavy traffic limit.