Exercise 1.1.2.

Hint: Use Lagrangian multipliers. The basic idea is that since it is iid then everything should be symmetry.

Exercise 1.1.3.

Pay attention the $\alpha_i$s in (a) are fixed. So they are not the variables you are going to optimize. Lagrangian multipliers will work, but if you define $Y_i = \sqrt{\alpha_i}X_i$, things will be better. For (b), just use

$$
\text{var}\left(\sum_i \alpha_i X_i\right) = \sum_i \alpha_i^2 \text{var}(X_i) + \sum_{i \neq j} \alpha_i \alpha_j \text{cov}(X_i, X_j)
$$

Exercise 1.1.12.

Go direct integration and the answer will come out. To show the finite moment you may need to control the tail with inequality

$$
\frac{1}{(1 + |x|)^k} \leq \frac{1}{|x|^k}
$$

Exercise 1.4.1.

Hint: Use the cdf and do a transformation then you will find what you need.

Exercise 1.4.2.
Hint: Cdf still works. For instance:

\[ P(-\log X < x) = P(X < e^{-x}) \]

Then compare the result and the cdf of an exponentially distributed variable.

**Exercise 1.4.13b.**

The same with the previous questions. Check Weibull distribution and its cdf in Wikipedia.

**Exercise class.1.**

Hint: Use equality

\[ \sum_i (X_i - \bar{X})^2 = \sum_i X_i^2 - n(\bar{X})^2 \]

and note that \( E(X_i^2) = \text{var}(X_i) + (\bar{X})^2 \). Do the same trick to find \( E(\bar{X})^2 \).

**Exercise class.2.**

This equation may make life easier:

\[ \sum_i (X_i - \bar{X})^2 = \sum_{(i,j)} \frac{1}{n} (X_i - X_j)^2 \]

The right hand side runs all combination of \((i, j)\). First prove the equation above, then carefully analyze your target using the formula for variances.

**Exercise class.3.**

For skewness, try to prove that for iid sample \( X_1, \ldots, X_n \), we have

\[ \mu_3(\sum X_i) = n\mu_3(X_i) \]

Here \( \mu_3 \) stands for third central moment.

For kurtosis, try to show that for iid sample \( X_1, \ldots, X_n \), we have

\[ \mu_4(\sum X_i) - 3(\mu_2(\sum X_i))^2 = n(\mu_4(X_i) - 3\mu_2(X_i)) \]

Here \( \mu_4 \) is 4th central moment and \( \mu_2 \) is 2nd central moment (variance).