
1. \[ s(4) = s(1) + \int_1^4 v(t) \, dt = 10 + 16.5 - 6.75 = 19.75 \]

2. \[ \int_0^{\pi/2} (\sin x + \cos x) \, dx = \left[ -\cos x + \sin x \right]_0^{\pi/2} = [-\cos(\frac{\pi}{2}) + \sin(\frac{\pi}{2})] - [-\cos(0) + \sin(0)] \\
= (0 + 1) - (-1 + 0) = 2 \]

3. \[ u = x^2, \quad du = 2x \, dx \quad \cdot \quad \int x e^{x^2} \, dx = \frac{1}{2} \int e^{x^2} (2x \, dx) = \frac{1}{2} \int e^u \, du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C \]

Alternate solution: \[ u = x^2 \] goes from \((-1)^2 = 1\) to \(1^2 = 1\) as \(x\) goes from \(-1\) to \(1\).

4. \[ u = 3 - x, \quad du = -dx \quad \cdot \quad \int \frac{1}{(3-x)^3} \, dx = -\int \frac{1}{(3-x)^3} (-dx) = -\int u^{-3} \, du = -\frac{1}{2} u^{-2} + C \\
= -\frac{1}{2} (x-3)^{-2} + C \]

5. \[ u = \ln x, \quad du = \frac{1}{x} \, dx \quad \cdot \quad \int \frac{\sqrt{\ln x}}{x} \, dx = \int u^{1/2} \, du = \frac{2}{3} u^{3/2} + C = \frac{2}{3} (\ln x)^{3/2} + C \]

6. \[ \frac{3}{x} = 1 \iff x = 3 \quad \bullet \quad \text{Figure A6} \bullet \]

\[ \text{[Area]} = \int_1^3 \left( \frac{3}{x} - 1 \right) \, dx = \left[ 3 \ln(x) - x \right]_1^3 = [3 \ln(3) - 3] - [3 \ln(1) - 1] = 3 \ln(3) - 2 \]

Figure A6
7. Base: Figure A7a • The cross section at $x$ is a square of width $w = e^x$. • Figure A7b •

$A(x) = w^2 = (e^x)^2 = e^{2x}$ for $0 \leq x \leq 1$ • [Volume] = \( \int_0^1 A(x) \, dx = \int_0^1 e^{2x} \, dx \) •

$u = 2x, \, du = 2 \, dx$ • \( \int e^{2x} \, dx = \frac{1}{2} \int e^{2x} (2x \, dx) = \frac{1}{2} \int e^u \, du = \frac{1}{2} e^u + C = \frac{1}{2} e^{2x} + C$ •

[Volume] = \( \left[ \frac{1}{2} e^{2x} \right]_0^1 = \frac{1}{2} e^2 - \frac{1}{2} e^0 = \frac{1}{2} \left( e^2 - 1 \right) \)

8. [Average value] = \( \frac{1}{2} \left[ \int_0^2 x^3 \, dx = \frac{1}{4} \left[ \frac{1}{4} x^4 \right]_0^2 = \frac{1}{4} \left( \frac{1}{4} (2^4) \right) - \frac{1}{4} \left( \frac{1}{4} (0^4) \right) = \frac{1}{4} (16) = 2 \) •

Figure A8 • The area of the region between the curve $y = x^3$ and the $x$-axis for $0 \leq x \leq 2$ is equal to the area of the rectangle. •

Or: The area of the region between the curve and the line $y = 2$ where $x^3 < 2$ is equal to the area of the region between the curve and the line $y = 2$ where $x^3 > 2$. 