Please simplify your answers to the extent reasonable without a calculator. Show your work. Explain your answers, concisely.

1. Two fair dice—one green, one red—each with sides numbered 1, . . . , 6, are rolled.
   a. [3 points] What is the probability that the green die shows an odd number? \( \frac{3}{6} = \frac{1}{2} \).
   b. [5 points] What is the probability that the sum of the numbers showing is 7 or 11? \( \frac{6 + 2}{36} = \frac{2}{9} \).
   c. [7 points] What is the probability that the green die shows an odd number and the sum of the numbers showing is 7 or 11? \( \frac{3 + 1}{36} = \frac{1}{9} \).
   d. [10 points] What is the probability that the green die shows an odd number or the sum of the numbers showing is 7 or 11? \( \frac{1}{2} + \frac{2}{9} - \frac{1}{9} = \frac{11}{18} \).

2. Suppose Karen Chen includes two triple Lutzs in her free skate program. The probability that she does a triple Lutz without falling is 0.9, unless it is the second of the program and she falls on the first one, in which case the probability drops to 0.7.
   a. [10 points] What is the probability that she does the second triple Lutz without falling? \( 0.9^2 + 0.1 \cdot 0.7 = 0.88 \).
   b. [15 points] Are the events “she does the first triple Lutz without falling” and “she does the second triple Lutz without falling” independent? Explain your answer. No, because \( 0.9 \cdot 0.88 \neq 0.81 \).

3. A (somewhat strange) person flips a fair coin 6 times. Each time the coin lands head up, the person takes a step forward; each time it lands tail up, the person takes a step back. Let \( -6 \leq X \leq 6 \) be a random variable indicating the position of the walker after the 6 coin flips, e.g., if the 6 flips are HHTHHT, then \( X = 2 \).
   a. [15 points] What is the probability distribution of \( X \)?
      \( X = H - T = 2H - 6 \), where \( H \) and \( T \) are the numbers of heads and tails, respectively. Notice that \( X \) takes only even values, and \( h = (x + 6)/2 \).
      \( P(X = 2k) = P(H = k + 3) = \binom{6}{k+3}/2^6 \).
   b. [5 points] What is \( \mathbb{E}[X] ? \) \( \mathbb{E}[X] = \mathbb{E}[2H - 6] = 2\mathbb{E}[H] - 6 = 2 \cdot 3 - 6 = 0 \).
   c. [10 points] What is \( \text{Var}[X] ? \) \( \text{Var}[X] = \text{Var}[2H - 6] = 4\text{Var}[H] = 4 \cdot 6(1/2)(1/2) = 6 \).

4. [20 points] As of today (when I’m writing this exam) the UCSD Men’s Soccer team has a record of 5 wins, 4 losses, 4 ties, and 1 game postponed. In how many ways could this have happened? \( \binom{14}{5,4,4,1} = 1260261 \).

5. [Extra credit: 25 points] Let \( A, B, C, D \) be random variables, each taking values in \( \{-1, 1\} \), i.e., \( P(A = 1) = p_A \), \( P(A = -1) = 1 - p_A \). Show that \( -2 \leq \mathbb{E}[AB] + \mathbb{E}[BC] + \mathbb{E}[CD] - \mathbb{E}[AD] \leq 2 \).
   [Hint: Use the linearity of expectation value.]
   \( \mathbb{E}[AB] + \mathbb{E}[BC] + \mathbb{E}[CD] - \mathbb{E}[AD] = \mathbb{E}[AB + BC + CD - AD] = \mathbb{E}[(A + C)B + (C - A)D] \).
   Only one of \( A + C \) and \( C - A \) is nonzero for any pair of values of \( A \) and \( C \), and the other is \( \pm 2 \), so the whole expression is \( \pm 2 \). Let \( 0 \leq p \leq 1 \) be the probability it is 2. Then \( \mathbb{E}[(A + C)B + (C - A)D] = 2p - 2(1 - p) = 2(2p - 1) \), which is between \(-2\) and 2.