Math 140B - Homework 6. Due Wednesday, May 29.

1. Rudin, Chapter 7, solve problems 20, 21, 23.

2. Show that power series can be integrated term by term within the radius of convergence. That is, assume that
\[ f(x) = \sum_{n=0}^{\infty} c_n x^n \]
has radius of convergence \( R > 0 \), and let \( -R < a < b < R \). Show that
\[
\int_{a}^{b} f(x) \, dx = \sum_{n=0}^{\infty} c_n \int_{a}^{b} x^n \, dx = \sum_{n=0}^{\infty} c_n \frac{b^{n+1} - a^{n+1}}{n+1}.
\]

3. Consider the series
\[ f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} x^{n+1}. \]

(i) Show that the radius of convergence is \( R = 1 \).
(ii) Differentiate term by term and confirm that for \( -1 < x < 1 \) we have
\[ f'(x) = \frac{1}{1+x}. \]
Use your knowledge of integration to conclude that
\[ f(x) = \ln(1 + x) \]
for \( |x| < 1 \).
(iii) Using the value \( x = 1 \), explain that
\[ \ln 2 = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}. \]
Make sure you justify this step correctly.

4. From the textbook, read Theorem 8.3, and write down a complete argument in your own words. Then, solve Rudin, Chapter 8, exercise 2.

5. Extra credit: Solve Rudin, Chapter 7, problem 25.