(1) Inhomogeneous second order equations.
   (i) General solution \( y = y_p + y_h \), where \( y_p \) is the particular solution, \( y_h \) is the homogeneous solution.
   (ii) Find a particular solution by undetermined coefficients
   \[ y'' + py' + qy = g(t). \]
   Three cases: \( g(t) \) can be exponential, trigonometric, polynomial.
   * For \( g(t) \) polynomial, look for \( y_p \) as a polynomial with undetermined coefficients. Try to guess its degree first.
   * For trigonometric \( g(t) \), look for \( y_p = A \cos t + B \sin t \).
   * For exponential case \( g(t) = e^{at} \), use
     \[ y_p = \frac{e^{at}}{f(a)} \]
     with \( f(a) = a^2 + pa + q \). If \( f(a) = 0 \), take
     \[ y_p = \frac{te^{at}}{f'(a)}. \]
   * For a term \( g(t) = e^{at} \times \) polynomial or trigonometric function , substitute \( y = e^{at}u \), find the differential equation for \( u \), then solve for \( u \) by undetermined coefficients.
   (iii) Alternatively, you may use variation of parameters
   \[ y = u_1(t)y_1(t) + u_2(t)y_2(t) \]
   where
   \[ u_1(t) = -\int \frac{y_2(t)g(t)}{W(y_1, y_2)} dt, \quad u_2(t) = \int \frac{y_1(t)g(t)}{W(y_1, y_2)} dt. \]

(2) First order systems of equations
   \[ \mathbf{x}' = A\mathbf{x}. \]
   (i) Find eigenvalues \( \lambda \) of \( A \):
   \[ \det(A - \lambda I) = 0. \]
   Eigenvectors \( \mathbf{v} \) are found by solving the system
   \[ (A - \lambda I)\mathbf{v} = 0. \]
   (ii) Finding solutions: find eigenvalues \( \lambda_1, \lambda_2 \) with eigenvectors \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \). General solution
   \[ \mathbf{x} = c_1 e^{\lambda_1 t}\mathbf{v}_1 + c_2 e^{\lambda_2 t}\mathbf{v}_2. \]
   (iii) Phase portraits. Distinct eigenvalues:
   * saddles (real eigenvalues of opposite sign)
   * nodes (sink or source) (real eigenvalues of same sign)
   * spiral (sink or source) (complex eigenvalues). To find the direction of spirals compute the velocity vector at a point on the trajectory.
   (iv) Repeated eigenvalues \( \lambda_1 = \lambda_2 = \lambda \). Defective case: one eigenvector \( \mathbf{v} \). Solutions
   \[ \mathbf{x}_1 = e^{\lambda t}\mathbf{v}, \quad \mathbf{x}_2 = e^{\lambda t}(\mathbf{w} + t\mathbf{v}) \]
   where
   \[ (A - \lambda I)\mathbf{w} = \mathbf{v}. \]
   Origin is an improper node.
(v) Fundamental matrix

\[ \Psi(t) = \begin{bmatrix} x_1 & x_2 \end{bmatrix}. \]

The Wronskian is the determinant of the fundamental matrix

\[ W(x_1, x_2) = \det \Psi(t). \]

The normalized fundamental matrix

\[ \Phi(t) = \Psi(t)\Psi(0)^{-1} = e^{At}. \]

Solution to IVP \( x' = Ax \) and \( x(0) = x_0 \) is

\[ x = e^{At}x_0. \]

(vi) Undetermined coefficients for systems.

(vii) Variation of parameters for systems

\[ x' = Ax + g(t). \]

A particular solution is given by

\[ x_p = \Psi(t) \int \Psi(t)^{-1} g(t) \, dt. \]

General solution is

\[ x = x_p + x_h \]

where \( x_h \) solves the homogeneous system.