Math 31CH - Homework 5. Due Friday, May 16.

Part I: Orientations.

1. Section 6.3: solve problem 6.3.7 and 6.3.11(b).

2. Section 6.3: solve problems 6.3.4, 6.3.6, 6.3.12.

3. Review Exercises for Chapter 6: solve problems 6.5, 6.6, 6.7(a).

4. Consider the surface $S \subset \mathbb{R}^4$ given by the equations:
   
   \[ x^2 + yz - yw = 1, \quad xy + zw + xw = 3. \]

   Orient $S$ using the differential form $\Omega = dz \wedge dw$.

   Write down a positive basis for the tangent space to $S$ at the point $(1, 1, 1, 1)$.

Part II: Integration over oriented manifolds.

5. Section 6.4: solve problems 6.4.4, 6.4.5, 6.4.6.

6. Integrate the form
   \[ \omega = x\,dx + y\,dy \]
   over the edges of the square with opposite vertices $(0,0)$ and $(1,1)$ directed counterclockwise.

7. Integrate the form
   \[ \omega = x\,dy \wedge dz + z\,dx \wedge dy \]
   over the plane region
   
   \[ 2x + y + z = 2, \quad x, y, z \geq 0 \]

   oriented by the upward pointing normal.

8. Integrate the form
   \[ \omega = x\,dy \wedge dz - y\,dx \wedge dz \]
   over the surface
   
   \[ z = 1 - x^2 - y^2, \quad z \geq 0 \]

   with the orientation given by the upward pointing normal.